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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME 80 JULY 1934 NUMBER 1

ULTRA-VIOLET STELLAR SPECTRA WITH ALUMINUM-COATED REFLECTORS

I. CORNELL EXPEDITION TO LOWELL OBSERVATORY

By S. L. BOOTHROYD

ABSTRACT

A short bibliography and historical résumé of the application of the evaporation process to astronomical mirrors is given. The personnel of the expedition consisted of R. William Shaw, Robley C. Williams, George B. Sabine, Dr. H. C. Ketcham, and the writer—all of Cornell University. Apparatus and observing conditions at the Lowell Observatory and at their mountain station are described. There were obtained 174 spectra of 97 stars, for the most part to the ultra-violet limit of atmospheric transmission.

A 10-inch concave mirror belonging to the physics department of Cornell University and the 15-inch mirror of the Lowell Observatory were coated with chromium, by Williams and Sabine early in the summer of 1932, for use by Messrs. Shaw, Williams, Sabine, and the writer in an attempt to photograph the extreme ultra-violet spectrum of the solar corona at the eclipse of August 31, 1932. Clouds prevented success in this undertaking.

In the fall of 1932 Messrs. Shaw, Williams, and Sabine[1] used the 10-inch chromium-coated mirror mentioned above, in combination with a small laboratory quartz spectrograph, both attached to the tube of the 12-inch equatorial telescope of the Fuertes Observatory of Cornell University, and obtained the ultra-violet spectrum of Vega almost to the limit of atmospheric transmission.

This encouraged us to consider seriously a project which had been in our minds since the early summer of 1932, namely, the organiza-

[1] *Astrophysical Journal*, **77**, 316, 1933.

tion of an expedition to a high-altitude observing station to secure the ultra-violet spectra of typical stars. The mountain station of the Lowell Observatory appealed to the writer as the best possible location for such work.

Director V. M. Slipher of the Lowell Observatory offered the use of the 15-inch mirror he had lent us for the eclipse and offered, as well, mountings for the 15-inch mirror both at the Lowell Observatory and at their mountain station. Besides this, Dr. Slipher made available the use of their laboratories and shop, as well as the use of a residence, with water, electricity, and fuel furnished.

Thus encouraged in our project, the writer, in March, 1933, applied to the National Research Council at Washington, D.C., and to the Heckscher Research Council of Cornell University for grants to cover traveling expenses and cost of building a stellar spectrograph and minor appurtenances thereto, as well as photographic supplies needed for the work. These funds were made available in the early summer of 1933.

HISTORICAL

There exists considerable misunderstanding regarding the invention and development of the evaporation process for use with astronomical mirrors. The first public notice of the evaporation technique was in 1890, at which time Edison was granted a patent on the process. Since that time numerous investigators have worked with it, including Harker,[2] Ritchl,[3] Richter,[4] Cartwright and Strong,[5] Cartwright,[6] and Williams and Sabine.[7]

In the summer of 1931 investigation was begun at Cornell University on the possibilities of an application of the process to large astronomical mirrors. This culminated in the successful evaporation of chromium,[8] and the subsequent coating of the Lowell Observatory 15-inch mirror with this metal in July, 1932. This mirror was used in the field at the total solar eclipse of August, 1932, and, to our knowledge, was the first large astronomical mirror to be coated by this process. Since this time considerable development has been

[2] *Proceedings of the Royal Society*, **88**, 522, 1933.

[3] *Zeitschrift für Physik*, **69**, 578, 1931.

[4] Richter has a British patent for the process.

[5] *Review of Scientific Instruments*, **2**, 189, 1931. [6] *Ibid.*, **3**, 289, 1932. [7] *Loc. cit.*

[8] *Science*, **76**, 67, 1932; *Popular Astronomy*, **40**, 405, 1932.

made at Cornell and particularly at the California Institute of Technology.[9]

At present aluminum is the best reflecting metal known in qualities of permanence and average reflectivity. The 15-inch mirror of the Lowell Observatory was aluminized for the expedition by Williams and Sabine in August, 1933. The 36-inch mirror of the Lick Observatory was aluminized in December, 1933, at the California Institute, and is the largest yet coated by this process.

PERSONNEL AND GENERAL PLAN OF WORK

Those co-operating with the writer on the expedition were R. William Shaw, Robley C. Williams, George B. Sabine, and Dr. H. C. Ketcham—all of Cornell University. Mrs. Alice B. Boothroyd and Mrs. Alice Ketcham accompanied the expedition to Flagstaff.

Director Slipher provided the exclusive use of the 15-inch reflector during the work at the Lowell Observatory. There is a shelter containing a clock-driven polar axis for this instrument, both at the main site of the Lowell Observatory, at an altitude of 7350 feet, and at the mountain station, which is situated on one of the higher peaks of the San Francisco mountains, almost exactly eight miles due north of the administration building, at an altitude of about 11,500 feet.

The plan of operations was to complete, adjust, and test the equipment at the shelter of the 15-inch telescope at the Lowell Observatory, as well as to get experience in its use where everything needed was available, and then to take the equipment to the mountain station and to observe there as long as weather conditions seemed propitious. If after that any time remained, work was to be resumed at the first location again. Following this plan, work was done in three periods, as follows:

First Period: At Lowell Observatory: August 29–September 8

Second Period: Observations at mountain station of Lowell Observatory, September 10–October 5

Third Period: Observations at Lowell Observatory, October 6–24, when the field work ended

The first period was used in completing and testing the equipment at the Lowell Observatory and in other necessary preparations.

[9] Strong, *Publications of the Astronomical Society of the Pacific*, **46**, 18, 1934.

THE APPARATUS

The stellar spectrograph, which was constructed in the shops of Cornell University and of the Lowell Observatory especially for this investigation, was a two-prism instrument. The optical parts consisted of a matched pair of Cornu quartz prisms having a refracting angle of 60° and a pair of crystalline quartz lenses having an aperture of $1\frac{7}{8}$ inches and an approximate focal length of 15 inches. The prisms were mounted on a base which could be rotated about an axis perpendicular to the refracting edges and yet could be securely clamped in any desired position. The prism base, the two lenses, an adjustable slit, and a plate-holder were attached appropriately to a triangular frame of duralumin. The frame was constructed from an I-beam with 2-inch face plates and 4-inch webb. This construction proved to be extremely rigid, there being no detectable effects of flexure for any position of the spectrograph while in operation. Moreover, the weight of the assembled instrument, without the case, amounted to only about 25 pounds.

All parts of the spectrograph, with the exception of the slit, were inclosed in a case made of three-ply wood. This case was the only provision made for maintaining an approximately constant temperature of the spectrograph.

It was found by experiment that for the short exposures used no further temperature control was necessary. The spectrograph was supported in the usual cradle on the side of the telescope-tube frame.

In the matter of photographic performance, the spectrograph was excellent. The definition was very good and a flat field was obtained from about λ 4500 to λ 2500. The dispersion is 39.0 A/mm at λ 3900 and 16.5 A/mm at λ 3000.

In order to calibrate the plates for their wave-length sensitivity and density-intensity relation, a standard lamp was used. This lamp, designed to operate on a 12-volt storage battery, at a known current, had a tungsten filament in a Corex A glass bulb. The lamp was carefully calibrated for spectral energy distribution in the laboratory before being used, and recalibrated afterward by the General Electric Laboratory at Nela Park. Intensity marks were placed on the plates by the aid of this lamp and a step-width diaphragm which was placed over the wide-open slit of the spectrograph.

The usual iron comparison spectrum was placed on either side of

the stellar spectrum, one exposure being made before the exposure on the star and the other after. The source of comparison spectra was a small iron spark operated by an induction coil. Sufficient inductance was used in the circuit to eliminate air lines and to give as nearly an arclike spectrum as possible. It was necessary to use a battery-operated spark for the comparison spectrum, inasmuch as there was no other source of electrical energy at the mountain station.

The iron spark gap and standard lamp, together with a quartz lens, were mounted on a support on the telescope tube in such a manner that light from either source could be made to form a suitable image on the slit of the spectrograph. To accomplish this a reflection from the secondary mirror of the telescope, but not from the primary mirror, was necessary. As a matter of fact, it would have been desirable to have the light from the standard lamp reflected also from the primary mirror, since this would have insured having stellar and standard light pass through the same optical path in the instrument. However, lack of suitable mechanical and optical parts made this difficult to obtain.

To insure a fairly uniform density wave-length distribution in the stellar spectrograms, a 1.4-mm Corex 986 A red-purple filter, of accurately known transmission characteristics, was placed over the camera lens. This filter was used only in those cases in which it was intended to analyze the plates with a microphotometer. The use of this filter did not unduly lengthen the exposure times. It was also used to equalize the wave-length density distribution in the intensity marks produced by the standard lamp.

Mention should be made of the fact that the secondary mirror is flat and that the focal ratio for the collimator of the spectrograph is only 1:10 while that of the primary mirror is 1:8, so that in the combination of mirror and spectrograph only 12 inches of the diameter of the primary mirror were used. As the 4-inch secondary mirror shadows the central area of the 15-inch, the surface which reflects light to the secondary is thus only a little larger than that of a 10-inch mirror.

DATA OBTAINED

Table I contains a list of the stellar spectrograms obtained in the three periods of observing. The total observing time was about 300 hours.

Besides these, four spectrograms of Saturn and one of Venus were obtained. The planets were at so low an altitude that atmospheric absorption prevented securing their extreme ultra-violet spectra. From three to nine exposures of the sun at different altitudes on each of fourteen days, for the study of atmospheric transmission, were obtained.

About two-thirds of the spectrograms are on Eastman 33 plates and the rest are on Cramer Iso-presto plates.

Plate I shows typical spectra of four different stars with iron comparison spectra.

TABLE I*

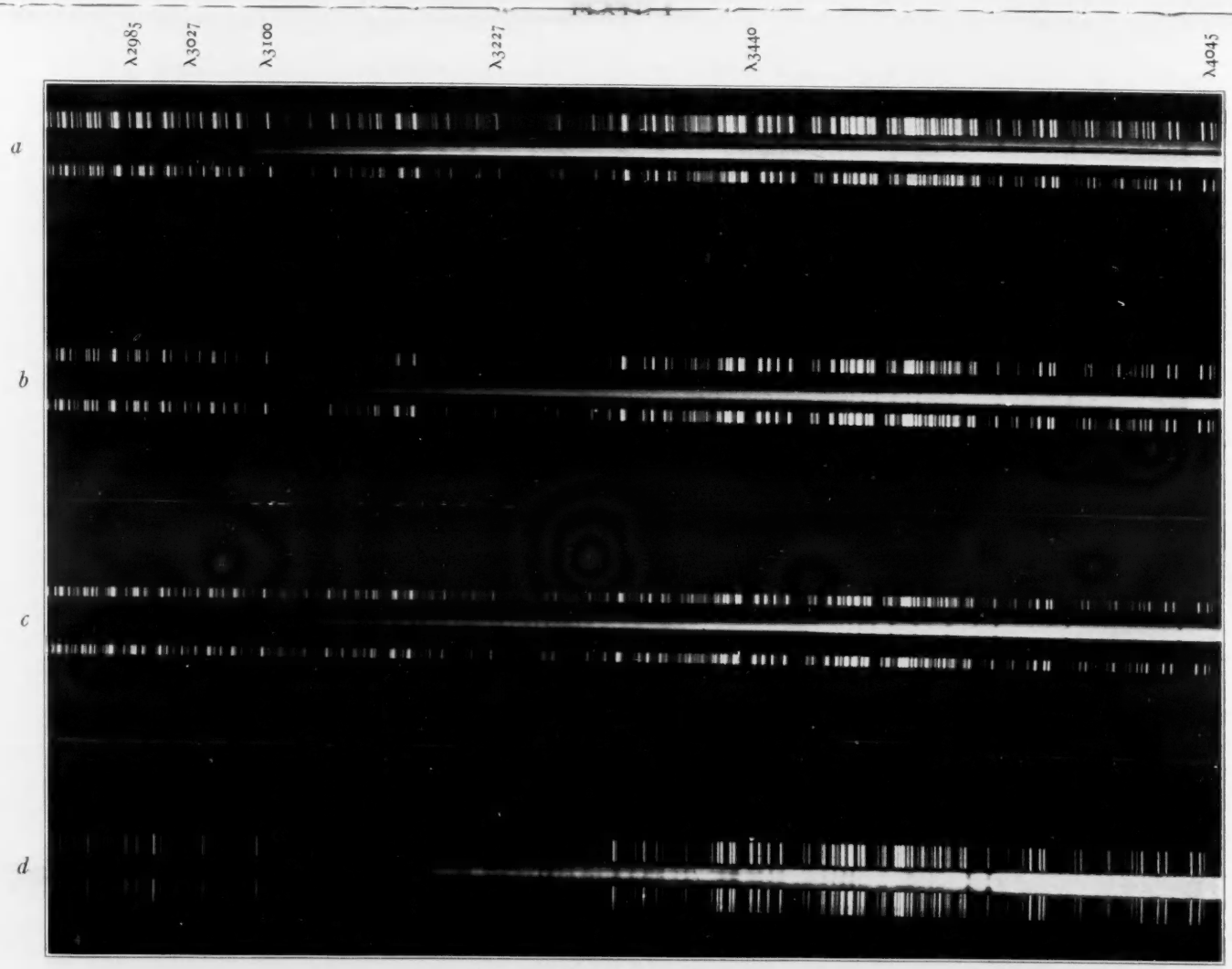
Observing Periods	Types of Stellar Spectra								Total
	O	B	A	F	G	K	M	Neb.	
I Test period at Lowell Observatory, September 2–8	.....	2	5	2	.....	3	3	.....	15
II At mountain station, September 11–October 4	3	31	26	5	3	5	.....	.....	73
III At Lowell Observatory, October 6–23	6	36	27	9	2	5	.....	1	86
Total	9	69	58	16	5	13	3	1	174

* Twenty-one of the stellar spectra were taken through the filter.

The work of measuring and interpreting the spectrograms is in progress; attention is being given to intensity distribution, line spectra of all types and relative abundance of elements in stellar atmospheres.

ACKNOWLEDGMENTS

The members of the expedition make the following acknowledgments for the aid which has made this project possible: To W. L. Putnam, trustee of the Lowell Observatory, to Dr. V. M. Slipher, and to the staff of the Lowell Observatory; to the National Research Council and to the Heckscher Research Council for their grants; to Professor Emeritus E. L. Nichols of Cornell University for the loan of the quartz prism; to Professor J. Papish of the chemistry department of Cornell University for the loan of the optical parts of the Steinheil quartz spectrograph; and to Mrs. Alice B. Boothroyd, who provided for the comfort of the members of the expedition and attended to much of the secretarial work.



TYPICAL ULTRA-VIOLET SPECTRA

a) Two spectra of ι Orionis; sp. Oe5, mag. 2.9. Exposures 6^m and 17^m.
b) κ Cassiopeiae; sp. Bo, mag. 4.2. Exposure 2^h3^m.
c) α Cygni; sp. A2p, mag. 1.3. Exposure 45^m.
d) α Aurigae; sp. Go, mag. 0.2. Exposure 20^m.

ULTRA-VIOLET STELLAR SPECTRA WITH ALUMINUM-COATED REFLECTORS

II. ENERGY-CURVES FROM λ 4350 TO λ 3000

By ROBLEY C. WILLIAMS

ABSTRACT

Apparatus and methods of obtaining spectra for the analysis of the energy-curves of early type stars from λ 4350 to λ 3000 are discussed. The value of using an ultra-violet filter for stellar spectra, for intensity marks, and for solar spectra, for the determination of atmospheric absorption is mentioned. Microphotometer-curves of typical spectra are reproduced.

Values determined for the atmospheric transmission above Flagstaff are given.

The Huggins bands of ozone are re-identified.

Stellar temperatures.—The method used for obtaining stellar temperatures from the energy-curves is outlined, and a table of the temperatures determined for fourteen stars is given.

Unusual energy-curves.—The energy-curves of three stars apparently having no continuous hydrogen absorption are shown.

Suggestions are made for methods to segregate the superposed effects of (1) hydrogen absorption, (2) possible deviation of stellar energy-curves from that of a black body, and (3) possible Rayleigh scattering.

APPARATUS

The apparatus and observing conditions for the investigation which resulted, in part, in the material for this paper are described in detail elsewhere in this issue.

Owing to differential atmospheric refraction, the image of a star formed by a reflector will be dispersed into a short spectrum. There are two possible ways of mounting a spectrograph to insure that light of all wave-lengths will reach the collimating lens, regardless of the zenith distance of the star. One method is to have the slit always in a vertical plane; the other is to employ a slitless spectrograph. Both methods necessitate rather more elaborate driving mechanism than was at our disposal. For this reason the slit was oriented parallel to the declination circles, but opened to a width of 0.1 mm. This width subtended an angle of about 10″ at the principal focus of the mirror, and computation[1] shows that for zenith distances not over 45° and barometric pressure of 23 inches the stellar image at λ 3000 is displaced from the one at λ 4250 by less than 2″. Although the visible image was used for guiding, it seems entirely permissible to assume

[1] Humphreys, *Physics of the Air*, p. 443, 1910.

that for small zenith distances there was no differential obstruction of λλ 3000–4250 by the slit-jaws.

A Corning Corex 986 A red-purple filter of 1.4-mm thickness was placed in the parallel beam in front of the camera lens in order to obtain the spectra, which are listed later. Spectra of the sun indicated that the filter placed in this position (on the advice of Dr. V. M. Slipher) did not affect the quality of the spectra. The transmission-curve of the filter is shown in Figure 1. A thinner filter of the

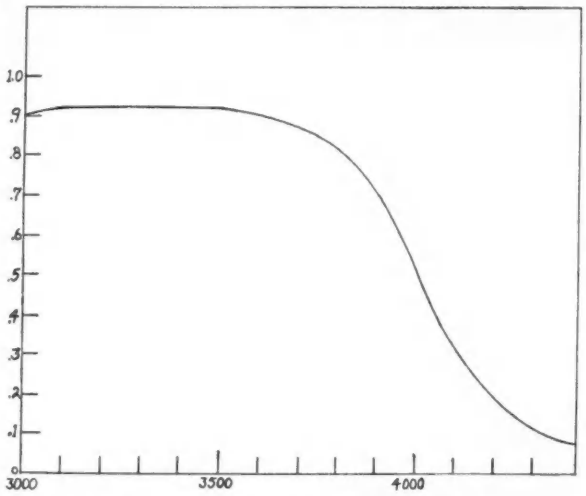


FIG. 1.—Transmission of 1.4-mm Corex 986 A red-purple filter

same kind would be advantageous in order to allow extension of the spectrum farther into the visible region.

A Corex A tungsten-filament lamp was operated in order to calibrate the plates for wave-length sensitivity and density-intensity relation. A step-width diaphragm was used over a wide slit, and the smallest width (0.133 mm) of the diaphragm openings was twice that at which intensity ceases to be proportional to slit-width.[2] The magnified image of the filament was focused on the slit by a thin quartz lens. The lamp was calibrated before being used, by visual comparison with a standard lamp on a spectrophotometer, at λ 4000 and λ 7000 with extrapolation to λ 3000. After the stellar research

[2] Nichols and Merrit, *Physical Review* (1st ser.), **31**, 502, 1910.

was finished the lamp was calibrated, through the courtesy of the General Electric Nela Park Laboratories, and the energy-curve is shown in Figure 2. The visual calibration agrees with this, within 10 per cent, from λ 4250 to λ 3100. The light from the intensity lamp passed through the same optical path as the light from the stars, except for the earth's atmosphere, the thin quartz focusing lens, and the reflection at the primary aluminum mirror.

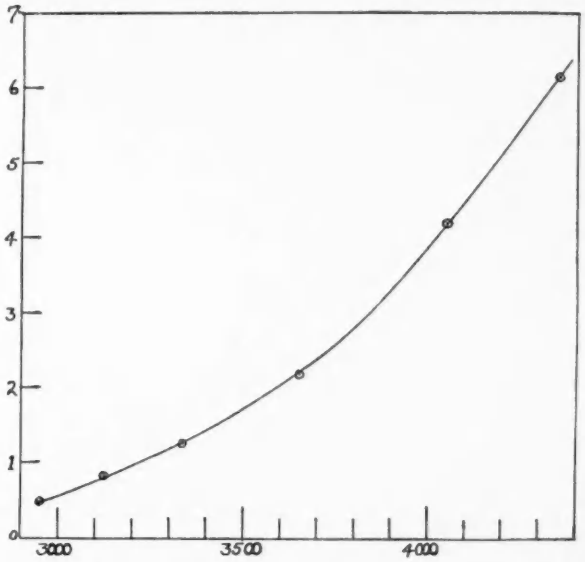


FIG. 2.—Spectral energy-curve of small Corex A tungsten-filament lamp

Eastman 33 and Cramer Iso-Presto plates were used. The time of development could not be standardized, for the length of exposure necessary to give proper density for microphotometric analysis was not known with great accuracy. The development continued until the density was judged visually to be sufficient. As will be shown later, standardized development would have been helpful, but not necessary.

ATMOSPHERIC TRANSMISSION

Photographs to determine the atmospheric transmission above Flagstaff and the San Francisco peaks were taken on the days of uninterrupted sunshine. The method followed was that described by

Fabry and Buisson.[3] Briefly, the procedure is to obtain the spectrum of the sun at various zenith distances throughout the day. A graph, plotted at any wave-length, of the logarithm of the intensity of the spectrum as a function of the secant of the zenith distance of the sun will be a straight line with a slope equal to the atmospheric absorption coefficient m. The transmission T for a given wave-length is found for the relation: $T = 1/\text{antilog}\ (m)$. The average result of

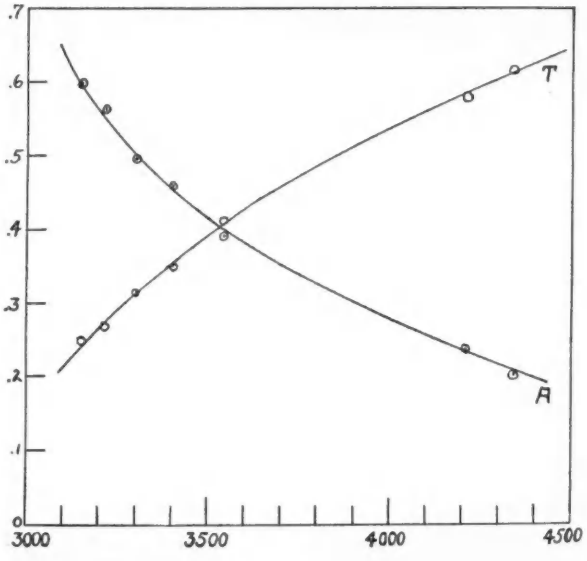


FIG. 3.—Atmospheric absorption and transmission determinations at Flagstaff. *A*, absorption; *T*, transmission.

the determinations made on the day preceding and the day between the two nights on which the stellar spectrograms were obtained with the filter is in Figure 3. The values obtained in the region of λ 4200 are lower than those obtained at a comparable elevation by Abbott[4] and Fowle.[5] It should be emphasized here that the determination of atmospheric transmission is of paramount importance in attempts to obtain highly accurate energy-curves as far as λ 3000. Our expedition was not as completely equipped in this respect as

[3] *Journal de physique et le radium*, **2**, 197, 1921.

[4] *Annals of the Astrophysical Observatory*, **4**, 193, 1922.

[5] *Smithsonian Physical Tables* (8th ed.), p. 608, 1933.

would have been possible, and to this extent the accuracy of the determination of absolute stellar temperatures has suffered. However, the relative values of the transmission at λ 3150 and λ 4250 obtained by us is in close agreement with published data for Mount

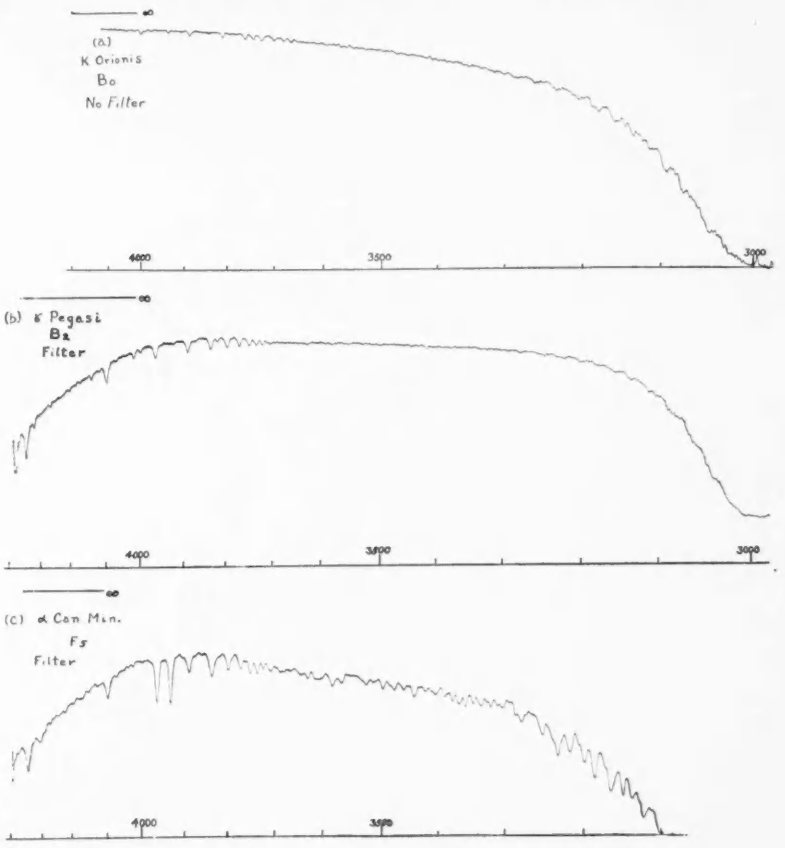


FIG. 4.—Stellar microphotometer-curves: (*a*) without filter; (*b*) and (*c*) with filter

Wilson.[6] Atmospheric transmission data for the station, at an elevation of 11,600 feet, have not yet been computed.

OZONE BANDS

Next to the Balmer series the most prominent feature of the microphotometer records in Figures 4 and 5 are the regularly occur-

[6] Pettit, *Publications of the Astronomical Society of the Pacific*, **46**, No. 269, p. 27, 1934.

ring bands extending from about λ 3450 to the ultra-violet end of the spectrum. These bands were first noticed by Huggins[7] in the spectrum of Sirius. Fowler and Strutt[8] definitely identified the bands discovered by Huggins in a stellar spectrum with the bands of ozone

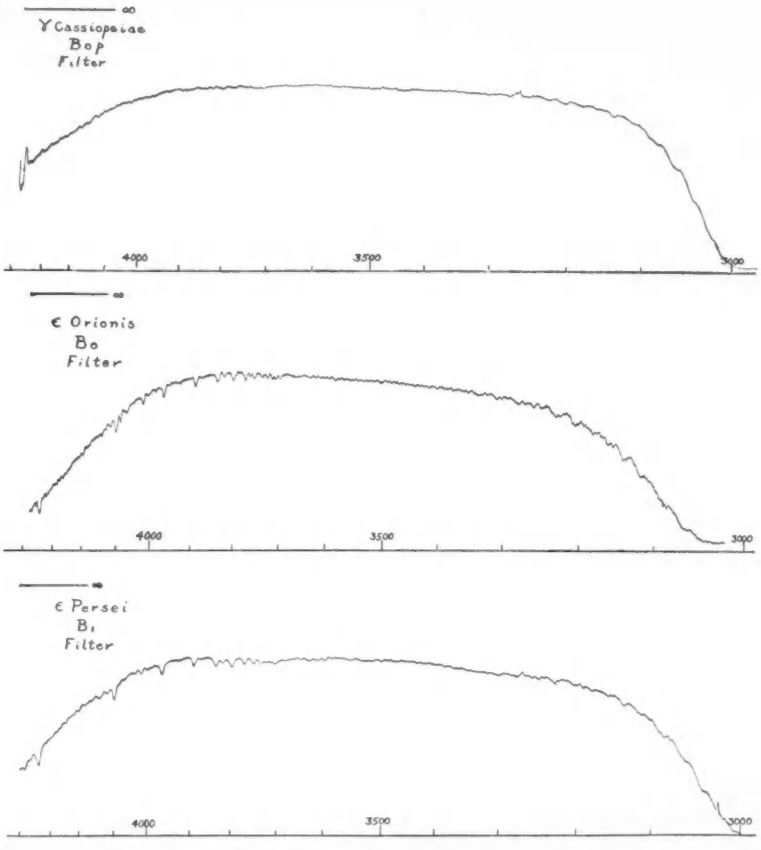


FIG. 5.—Stellar curves apparently showing no continuous hydrogen absorption

obtained in the laboratory. The present investigation confirms the identifications of the Huggins bands, and adds several bands on the ultra-violet side of these. A paper concerning the wave-lengths and identifications of these bands will appear soon.

[7] *Proceedings of the Royal Society*, **48**, 216, 1890.

[8] *Ibid.*, **93**, 577, 1917.

MICROPHOTOMETER-CURVES

An example of a microphotometer-curve of a spectrum taken without a filter is shown in Figure 4*a*. Although the spectrum was photographed at the mountain station, the λ 4200 region had to be heavily exposed in order to obtain usable intensity at λ 3050. Figure 4*b* shows a record of a spectrum of a star of approximately the same temperature, taken with the filter. Measurable intensity is obtained to λ 3010 without overexposure of the plate at any wave-length. Figure 4*c* is the curve obtained from a star of much lower temperature, and consequently the spectrum extends only to λ 3100. Spectra of the K- and M-type stars were not taken with a filter, for no attempt was made by us to obtain their temperatures photometrically. A discussion by R. W. Shaw of their absorption lines is to appear later.

TEMPERATURE DETERMINATIONS

Stellar temperatures can be determined by the application, at two wave-lengths, of Wien's approximation to Planck's law:

$$E_\lambda = C_1 \lambda^{-5} e^{\frac{-C_2}{\lambda T}},$$

where C_2 has the value 1.433 cm-deg. The expression for the temperature then becomes

$$T = \frac{1.433\left[\frac{\lambda_2 - \lambda_1}{\lambda_2 \lambda_1}\right]}{\log_e \frac{E_{\lambda_2}}{E_{\lambda_1}} - 5 \log_e \frac{\lambda_1}{\lambda_2}} \text{ in degrees absolute.} \qquad (\lambda_2 > \lambda_1)$$

The determination of the relative energies at two wave-lengths is, consequently, the problem. The energy leaving the star within a small interval $d\lambda$, at a wave-length λ_1, is affected by selective absorption in the stellar atmosphere, possible space-reddening, telluric atmospheric absorption, mirror reflectivity, spectrograph and filter absorption, prismatic dispersion, and plate sensitivity. The second possibility is not considered in this preliminary paper. The last three are effectively eliminated by the use of the intensity lamp. The first

effect is treated in the following way. From curves similar to Figures 4 and 5, a density-curve of the continuous spectrum is obtained. This curve will indicate the amount of continuous hydrogen absorption at the end of the Balmer series. By an extrapolation of the energy-curve from λ 4350 to λ 3750 over the region of the continuous absorption, giving the extrapolated curve the same shape as the energy-curve below it, an idea is obtained of the true energy-curve without the hydrogen absorption.

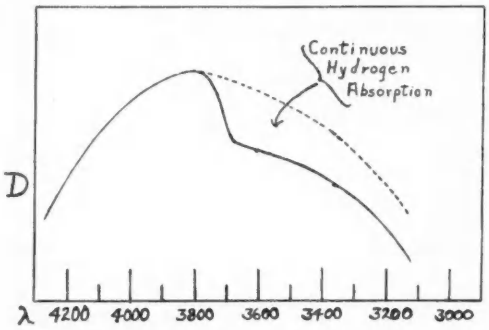


FIG. 6.—Method of correction for continuous hydrogen absorption.

The procedure is illustrated in Figure 6, and is the same as that followed by Yü[9] at the Lick Observatory. This method assumes that the continuous hydrogen absorption is constant as far as λ 3100, and a suggested method for testing this is discussed later.

The apparent intensity P_{λ_1} on the plate of the stellar spectrum at any wave-length may be written

$$P_{\lambda_1} = E_{\lambda_1} A_{\lambda_1} R^2_{\lambda_1} S_{\lambda_1} , \qquad (1)$$

where E_{λ_1} is the energy of the star spectrum, A_{λ_1} is the atmospheric transmission, $R^2_{\lambda_1}$ is the reflectivity of the two aluminum mirrors, and S_{λ_1} is a factor involving filter transmission, spectrograph absorption and dispersion, and plate sensitivity.

For the intensity lamp (reflected from one mirror) at the same wave-length one may write

$$L_{\lambda_1} = I_{\lambda_1} R_{\lambda_1} S_{\lambda_1} , \qquad (2)$$

where I_{λ_1} is the lamp intensity at wave-length λ_1. (Computation shows that the differential losses by reflection and absorption of the thin focusing lens from λ 4250 to λ 3000 are negligible.)

[9] *Lick Observatory Bulletin*, **12**, 104, 1926.

If two wave-lengths λ_1 and λ_2 of equal density on the stellar spectra are chosen (subject to the continuous absorption spectrum mentioned above), we can write

$$\frac{P_{\lambda_1}}{P_{\lambda_2}} = 1 = \frac{E_{\lambda_1}}{E_{\lambda_2}} \cdot \frac{A_{\lambda_1}}{A_{\lambda_2}} \cdot \left(\frac{R_{\lambda_1}}{R_{\lambda_2}}\right)^2 \frac{S_{\lambda_1}}{S_{\lambda_2}}. \tag{3}$$

For the intensity lamp at the two wave-lengths we can write

$$\frac{L_{\lambda_1}}{L_{\lambda_2}} = \frac{I_{\lambda_1}}{I_{\lambda_2}} \cdot \frac{R_{\lambda_1}}{R_{\lambda_2}} \cdot \frac{S_{\lambda_1}}{S_{\lambda_2}}. \tag{4}$$

All the factors except S_{λ_1} and S_{λ_2} are known or can be measured. Consequently the ratio $S_{\lambda_1}/S_{\lambda_2}$ can be determined and substituted in equation (3). Equation (3) becomes

$$\frac{E_{\lambda_2}}{E_{\lambda_1}} = \frac{A_{\lambda_1}}{A_{\lambda_2}} \cdot \frac{R_{\lambda_1}}{R_{\lambda_2}} \cdot \frac{I_{\lambda_2}}{L_{\lambda_2}} \cdot \frac{L_{\lambda_1}}{I_{\lambda_1}}.$$

The atmospheric transmission factors are determined by multiplying the vertical transmission by the secant of the zenith distance. The reflectivity of the primary mirror was taken as 80 per cent at λ 3100 and 85 per cent at λ 4250. The ratios $I_{\lambda_1}/L_{\lambda_1}$ and $I_{\lambda_2}/L_{\lambda_2}$ were determined from the intensity marks. The density-intensity relations were obtained for each plate and were found to be very closely the same at $\lambda\lambda$ 3100–4250 for any one plate. Of course, owing to lack of standardized development, this contrast factor varied widely from plate to plate.

A table is given on the following page for the fourteen stars containing (1) the wave-lengths at which the corrected spectra were of equal apparent intensity, (2) the ratio I_λ/L_λ for these wave-lengths (in arbitrary units), (3) the value of the atmospheric absorption m, (4) the zenith distance, (5) the true stellar intensities (in arbitrary units), and (6) the temperatures.

UNUSUAL ENERGY-CURVES

Of the early type spectra taken with the filter, three were found to possess unusual intensities at the end of the Balmer series. Figure 5

shows the microphotometer-curves of the stars ϵ Persei, ϵ Orionis, and γ Cassiopeiae. In the first two the continuous intensity between

TABLE I

Star	Spectral Class HD	Right Ascension	(1) λ	(2) I_λ/L_λ	(3) m	(4) Zenith Distance	(5) E_λ	(6) $T°$ Abs.
10 Lacertae	Oe5	22^h36^m	4100	0.845	.25	7°	2.63	40,000
			3250	0.725	.525	7	6.66	
γ Cassiopeiae	Bop	0 53	4180	1.02	.235	30	2.28	20,000
			3100	1.50	.64	30	5.28	
ϵ Orionis	Bo	5 33	4200	1.17	.225	40	2.14	20,000
			3100	2.56	.635	40	5.00	
ϵ Persei	B1	3 53	4250	1.25	.235	11	1.70	21,000
			3110	1.42	.625	11	4.46	
γ Pegasi	B2	0 10	4180	0.92	.225	24	1.23	21,000
			3120	1.79	.64	24	3.34	
γ Orionis	B2	5 21	4250	1.25	.225	29	1.82	15,000
			3100	2.50	.645	29	3.20	
ζ Tauri	B3p	5 34	4240	2.64	.225	15	2.03	13,000
			3100	4.40	.64	15	3.75	
η Tauri	B5p	3 43	4250	1.32	.225	11	1.57	10,000
			3180	2.86	.58	11	2.00	
β Tauri	B8	5 22	4250	1.03	.225	18	2.1	10,000
			3150	2.02	.625	18	3.14	
β Orionis	B8p	5 11	4250	0.725	.225	42	3.30	9,000
			3270	2.28	.51	42	3.20	
γ Geminorum	Ao	6 34	4250	1.09	.225	19	1.98	10,000
			3130	1.98	.615	19	3.28	
α Cygni	A2p	20 39	4250	1.55	.225	10	1.33	11,000
			3140	3.86	.605	10	1.03	
α Aquilae	A5	19 47	4250	0.91	.225	34	2.54	8,000
			3250	2.90	.525	34	2.26	
α Canis Minoris	F5	7 36	4250	0.91	.225	31	2.46	6,500
			3240	3.23	.53	31	1.85	

the Balmer lines decreases as the limit of the series is reached, and then resumes its original value at about λ 3650 for ϵ Persei and λ 3700 for ϵ Orionis. Although γ Cassiopeiae has no Balmer absorp-

tion lines, its continuous energy-curve shows an increase at λ 3650 similar to that for ε Persei. These three stars are very hot, and it is probable that their light does not undergo continuous hydrogen absorption. Later detailed work will attempt to ascertain this.

SUGGESTIONS

The possible differential loss of stellar light due to Rayleigh scattering in space is very difficult to detect, for in the ultra-violet region, where the scattering effect is most pronounced, there are generally the superposed effects of continuous hydrogen absorption and possible deviation from a black-body curve. In order to determine the magnitude of the first effect with any certainty, careful measurements must first be made of the latter effects. A small number of stars whose energy-curves are like those of ε Orionis, ε Persei, and γ Cassiopeiae, and whose positions in space are such as to make Rayleigh scattering very unlikely, might be selected as standard stars for measurement. Energy-curves of these stars could be determined with extreme care from the infra-red to λ 3000, using the standard stars themselves to obtain the atmospheric transmission. As the microphotometer-curves have shown, the spectra of early type stars are continuous in the far ultra-violet and do not have the strong absorption lines and bands of the solar spectrum. Further, the distribution with wave-length of the energy that gets through the earth's atmosphere is considerably more uniform for an early type star than for the sun. Therefore, such stars are most suitable for determining the atmospheric transmission at short wave-lengths. With a few standard stars so measured the usual procedure of comparison with other stars at equal zenith distances could be followed in order to obtain secondary standards. Variations of continuous hydrogen absorption with wave-length and with spectral type could then be determined absolutely for a number of stars in selected, non-scattering regions. Only after considerable data concerning these variations are obtained is it possible to look for Rayleigh scattering with certainty. The usual comparison could then be made of the light of a star which has passed through a scattering region with that of another star of the same type which has not

passed through such a region. Suitable ultra-violet transmitting filters can be used to partly equalize the intensities of the standard star and the abnormally red star throughout the spectrum. At present this equalization either has not been attempted or has been effected by choosing two stars of different spectral class.[10]

PHYSICS DEPARTMENT
CORNELL UNIVERSITY
March 16, 1934

[10] Struve, Keenan, and Hynek, *Astrophysical Journal*, **79**, 1, 1934.

COMPARISON OF THE PASCHEN AND THE BALMER SERIES OF HYDROGEN LINES IN STELLAR SPECTRA*

By PAUL W. MERRILL and OLIN C. WILSON, JR.

ABSTRACT

Introduction.—The structure of the hydrogen spectrum and previous observations of the Paschen series are briefly described. The behavior of the Paschen lines $m=12$ to $m=24$, recently photographed at Mount Wilson in various types of stellar spectra, resembles in general that of the Balmer lines in the same types. The Paschen series is very conspicuous in the *c* stars β Orionis and α Cygni.

Photometric data.—Measurements of structure and intensity have been made by the usual photometric methods on numerous lines in both hydrogen series in the spectra of α Leonis B8n, α Lyrae A0, α Canis Majoris A0, β Orionis cB8, and α Cygni cA2.

Structure of lines.—Most lines not affected by the overlapping of neighboring lines have contours of the simple exponential form, although in α Leonis the centers are flattened. In the broad-line stars α Leonis and α Lyrae considerable departure from the exponential form is exhibited by the wings of lines near the heads of the series. In β Orionis, α Cygni, α Leonis, and in the emission-line stars γ Cassiopeiae and P Cygni the shapes of Balmer and Paschen lines are similar, with the dimensions proportional to wave-length; but this is not true of α Lyrae, in whose spectrum the Balmer lines have relatively more intense wings.

Central intensities.—Measured values for numerous lines are given in Table VI. In general the central intensities of Paschen lines are less than those of the corresponding Balmer lines. In α Leonis the Paschen series is relatively intense and has a slow decrement. In β Orionis and α Cygni both series exhibit slow decrements and little overlapping.

Problem of overlapping lines.—The shapes as well as the central intensities of many lines, particularly in the Balmer series, are probably seriously modified by the overlapping wings of adjacent lines. The problem is to find the shapes and intensities of the "true" or original lines which by their mutual interaction produce the observed curve. A physically reasonable sequence of lines has been found which yields close approximations to the observed shapes and intensities in α Lyrae but gives too great a reduction of the general intensity of the whole spectrum toward the head of the series. This difficulty, which appears to be of a rather general nature, is briefly discussed.

Total line absorption.—Measured values for lines of both series in α Cygni, β Orionis, and α Leonis are given in Table VII. From these data Unsöld's equation gives the numbers of atoms in the second and third levels. The numbers computed for a given level from various lines differ systematically, indicating that the theory is incomplete. The maximum numbers, used in Boltzmann's equation for thermal equilibrium, give reasonable values of the temperature of the absorbing hydrogen.

Intensities of emission lines.—Several bright lines in both series were measured in γ Cassiopeiae and P Cygni. The relative importance of induced and spontaneous emission is discussed. The photospheric temperatures computed from the intensities of pairs of lines having a common upper level depend on the assumed distribution of the atoms with respect to azimuthal quantum number. An assumed distribution corresponding to thermal equilibrium leads to reasonable temperatures for both stars. The possibility of determining the amount of space absorption by comparison with other data is pointed out.

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 494.

INTRODUCTION

According to the quantum theory, a hydrogen atom must reside in one of the so-called stationary states, whose negative energies are inversely proportional to the squares of integers from 1 to infinity. A series of spectral lines is formed by transitions between any level and all the upper levels. Transitions from higher levels to each of the first five levels have been observed in the laboratory and are known by the names indicated in Table I. The Lyman series lies entirely in the far ultra-violet, inaccessible in stellar spectra because of the heavy absorption of the earth's atmosphere, while the Brackett and the Pfund series are deep in the infra-red where present methods are inadequate to record lines in stellar spectra. The Balmer series is favorably situated, and, since the pioneer observations of Sir

TABLE I

THE VARIOUS SERIES OF HYDROGEN LINES

Lower Level	Name	Location in Spectrum
$m=1$	Lyman	1,200– 900 A
2	Balmer	6,600– 3,600
3	Paschen	19,000– 8,200
4	Brackett	41,000–15,000
5	Pfund	74,000–23,000

William Huggins, has been very extensively studied in the spectra of astronomical bodies. Recent improvements in photographic emulsions for the near infra-red have made it feasible to photograph stellar spectra to about 9000 A, and thus to record a portion of the Paschen series. The positions with respect to regions of atmospheric absorption of individual lines of both series are shown in Figure 1. The general behavior of the Balmer lines in both laboratory and celestial sources is too well known to require description here, but it may be useful to summarize previous observations of the Paschen lines.

Laboratory observations of the first two lines,[1] $m=4$, λ 18,751 A

[1] Throughout this *Contribution* each hydrogen line will be designated by the total quantum number, m, of its upper state. The symbol for the Balmer series will be H, for the Paschen series P.

and $m = 5$, λ 12,818 A, by Paschen himself,[2] using a bolometer, were reported in 1908. About fourteen years later, F. S. Brackett,[3] working at Johns Hopkins University with a sensitive thermocouple constructed by A. H. Pfund, found three or four additional lines. The first photographs of Paschen series lines were made a few years later at Johns Hopkins by A. H. Poetker[4] on neocyanin sensitized plates prepared by the Eastman Kodak Company. The lines P7 λ 10,049 to P11 λ 8863 were included.

The first astronomical identification was of the line P7 λ 10,049 by H. D. Babcock on his photographs of the solar spectrum[5] and on the bolometric tracings of Abbot and Freeman.[6] Babcock also sus-

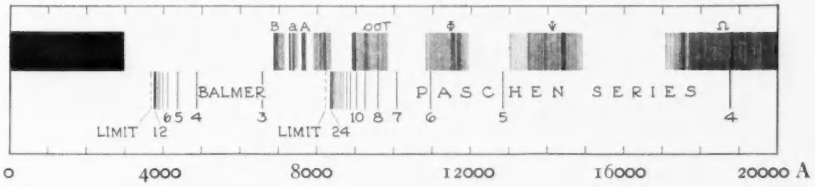


FIG. 1.—Lines in the two series of hydrogen and the chief absorption bands in the earth's atmosphere.

pected the presence of the preceding line, λ 10,938, on the bolometric record. Recently H. W. Babcock[7] found nine members of the Paschen series, P8 λ 9546 to P16 λ 8502, inclusive, as diffuse emission lines on spectrograms of the solar chromosphere made without an eclipse at the 150-foot tower telescope of the Mount Wilson Observatory. The first stellar spectrograms to show the Paschen lines were taken by one of the writers with the 100-inch telescope in June, 1932.[8] All the infra-red spectrograms used in the present study were taken between that date and July, 1933. An account of the methods and results of this preliminary investigation of the infra-red, with detailed observational data, will be found in *Contribution* No. 486.[9]

[2] *Annalen der Physik*, **27**, 537, 1908. [3] *Ap. J.*, **56**, 154, 1922.

[4] *Phys. Rev.* **30**, 418, 1927.

[5] C. E. St. John and Others, "Revision of Rowland's Preliminary Table of Solar Spectrum Wave-Lengths," *Carnegie Institution of Washington Pub.*, No. 396, 1928.

[6] *Smithsonian Misc. Coll.*, **82**, No. 1, 1929.

[7] *Pub. A.S.P.*, **44**, 323, 1932.

[8] *Ibid.*, p. 269, 1932. [9] *Ap. J.*, **79**, 183, 1934.

Table II gives the wave-lengths of the Balmer lines tabulated by A. Fowler[10] and the wave-lengths of corresponding Paschen lines computed from these values.

TABLE II

WAVE-LENGTHS OF BALMER AND PASCHEN LINES OF HYDROGEN

Upper Level m	Balmer Series			Paschen Series	
	H	I A	f	I A	f
3	α	6562.79	0.641	.....	.....
4	β	4861.33	.119	18751.05	0.842
5	γ	4340.47	.045	12818.11	.151
6	δ	4101.74	.022	10938.12	.056
7	ϵ	3970.08	.013	10049.39	.028
8	ζ	3889.05	.0080	9545.98	.016
9	η	3835.39	.0054	9229.03	.0102
10	θ	3797.90	.0038	9014.91	.0069
11	ι	70.63	.0028	8862.79	.0048
12	κ	50.15	.0021	8750.47	.0036
13	λ	34.37	.0017	8665.03	.0027
14	μ	21.94	.0014	8598.40	.0021
15	ν	11.97	.0011	8545.39	.0017
16	ξ	3703.86	.00088	8502.50	.0014
17	o	3697.15	.00072	8467.27	.0012
18	π	91.56	.00061	8437.96	.00097
19	ρ	86.83	.00052	8413.33	.00083
20	σ	82.81	.00044	8392.41	.00071
21	τ	79.36	.00038	74.49	.00061
22	υ	76.36	.00033	59.01	.00053
23	ϕ	73.76	.00029	45.55	.00047
24	χ	71.48	.00026	33.79	.00041
25	ψ	69.47	.00023	23.43	.00036
26	ω	67.68	0.00020	14.27	0.00032
27	.....	66.10	.....	8306.11	.....
28	.....	64.68	.....	8298.84	.....
29	.....	63.40	.....	92.31	.....
30	.....	62.26	.....	86.43	.....
31	.....	61.22	.....	81.14	.....
32	.....	60.28	.....	76.33	.....
33	.....	59.42	.....	71.95	.....
34	.....	58.64	.....	67.95	.....
35	.....	57.93	.....	64.29	.....
36	.....	57.27	.....	60.94	.....
37	.....	56.67	.....	57.86	.....
.....	.....	.....	.....	.....	.....
∞	.....	3645.98	.....	8203.58	.....

The appearance and general behavior of the Paschen absorption lines in the various types of the stellar sequence as judged by the

[10] *Report on Series in Line Spectra*, London: Fleetway Press, 1922.

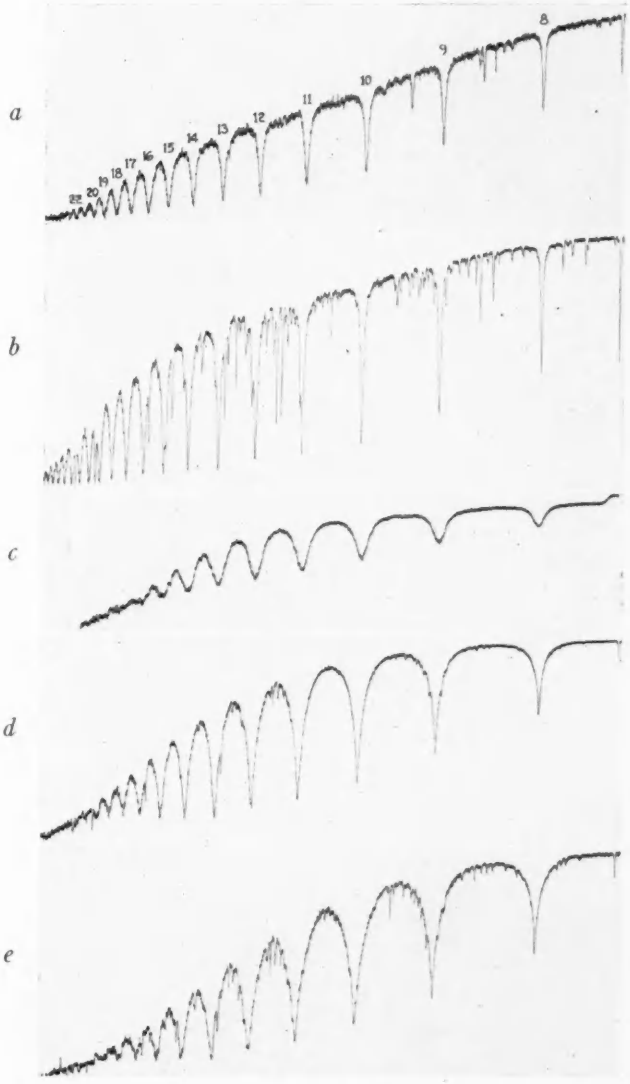


FIG. 2.—Microphotometer tracings of the Balmer series

a)	β Orionis	V324
b)	α Cygni	343
c)	α Leonis	317
d)	α Lyrae	343
e)	α Canis Majoris	334

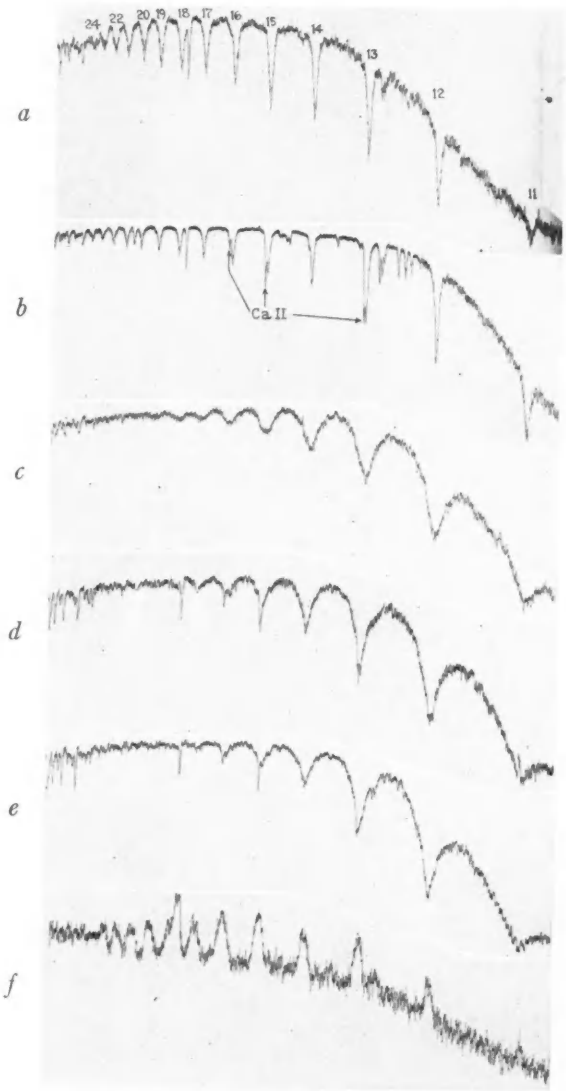
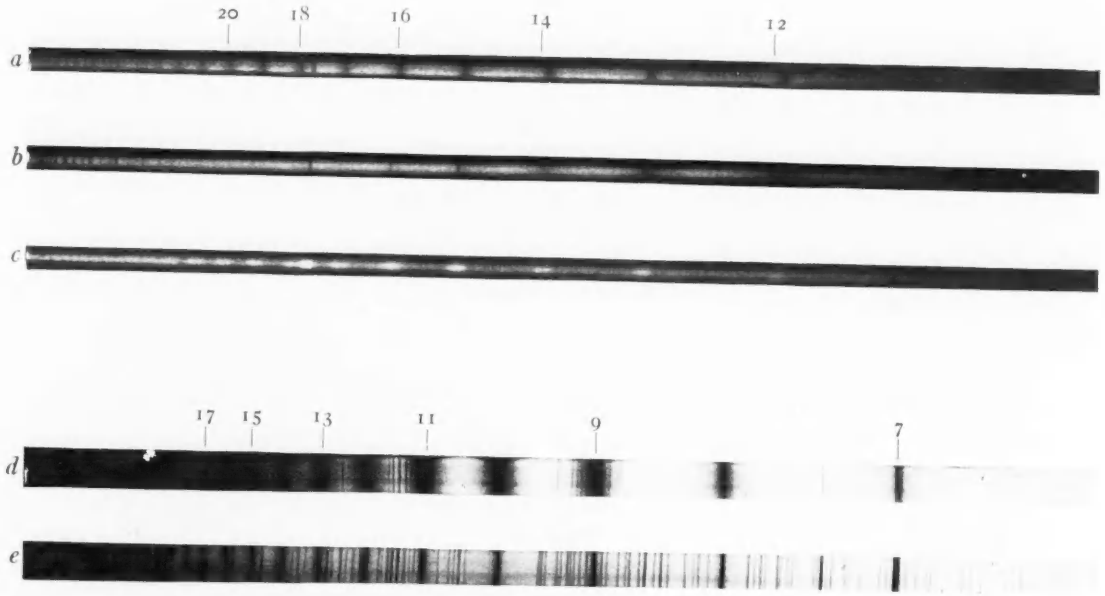


FIG. 3.—Microphotometer tracings of the Paschen series

a)	β Orionis	G742
b)	α Cygni	776
c)	α Leonis	744
d)	α Lyrae	757
e)	α Canis Majoris	737
f)	γ Cassiopeiae	578

PLATE II



TYPICAL PHOTOGRAPHS OF HYDROGEN SERIES IN STELLAR SPECTRA

Paschen Series		*Balmer Series*	
a) β Orionis	G742	*d*) α Lyrae	V343
b) α Canis Majoris	G737	*e*) α Cygni	V343
c) γ Cassiopeiae	G578		

present photographs, which show the lines from $m = 11$ to $m = 24$, are similar to those of the Balmer lines. This fact may be seen from the illustrations in Plate II, as well as from those in Plate I of *Contribution* No. 486; from the photometric tracings in Figures 2 and 3; and from the data in Table III, which indicate the last visible lines of both series in the spectra of several stars of types B and A. In the spectrum of Procyon, dF5, the Paschen series is inconspicuous but may be seen faintly from $m = 11$ to $m = 17$ or 18. The decrement of intensity from

TABLE III

LINES OF SHORTEST WAVE-LENGTH OBSERVED IN BALMER AND PASCHEN SERIES

Star, Type	Balmer Series		Paschen Series	
	m		*m*	
α Virginis	15	Strong	15	Present
B2n	16	Probably present as a blend with *He* 3705	16	Doubtful
α Leonis	16	Distinct	19	.....
B8n	17	Doubtful	.....	.....
β Orionis	23	.....	24	Fairly well marked*
cB8	.....	.....	.....	.....
α Lyrae	19	Well marked	18	.....
Ao	20	Doubtful	.....	.....
α Canis Majoris	18	.....	17	Present
Ao	.....	.....	18	Probable
α Cygni	28	Fairly well marked	24	Strong and sharp
cA2	29	Doubtful	25	Probable, but atmospheric lines interfere
	30		26	

* Following lines, if present, are lost among atmospheric lines.

line to line is small. In γ Cygni, cF8, the lines are stronger and better defined and exhibit an extremely small decrement. They may be traced from $m = 11$ to $m = 22$ or 23. In the ultra-violet spectra of these F-type stars, especially in that of γ Cygni, the closely packed metallic lines make it difficult to recognize weak members of the Balmer series; in Procyon the series appears to terminate with $m = 17$, while in γ Cygni it may possibly continue farther in low intensity. In the later spectral types both hydrogen series are of relatively low intensity, and present observations do not afford a basis for com-

parison. H. D. Babcock's photographs of the infra-red solar spectrum appear to show that the line P7 is more intense than the corresponding Balmer line $H\epsilon$.

Three stars in whose spectra certain members of the Balmer series appear as emission lines have been photographed in the infra-red. In γ Cassiopeiae, Bone, and P Cygni, Beq, numerous Paschen lines are also bright; but in R Hydrae, M7e, no Paschen lines, either bright or dark, have been distinguished.

PHOTOMETRIC DATA

Previous photometric investigations of hydrogen lines in stellar spectra[11] have dealt largely with the first three or four lines in the Balmer series.[12] The present *Contribution* gives results for higher members of this series, and for the first time makes available comparable data for the Paschen series.

The Mount Wilson spectrograms of the Balmer series, made on the Eastman 33 emulsion with a three-prism "ultra-violet" spectrograph and 15-inch camera have a dispersion at λ 3771, H11, of 12.5 A/mm; at λ 3671, H24, of 10.4 A/mm. The width of the stellar spectrum is about 1.0 mm. For photometric calibration a raster and an auxiliary lamp permit ten standard spectra to be impressed on the plate during a stellar exposure.[13] The spectrograms of the Paschen series, made with a plane-grating spectrograph and 18-inch camera, have a dispersion of 33.4 A/mm.[14] The width of the spectrum is about 0.3 mm. Photometric standards were obtained by exposures with a tube sensitometer through a deep-red filter. The two methods of standardizing, tested on ordinary emulsions, were found to give consistent results.

The stars investigated, with the designations of the plates from

[11] A useful list of references is appended to a paper by E. G. Williams, "Stellar Hydrogen Line Contours and Their Variation with Temperature and Surface Gravity," *Annals of the Solar Physics Observatory, Cambridge*, **2**, 25, 1932.

[12] Miss C. H. Payne, *Proc. Nat. Acad.*, **14**, 399, 1928, and S. Günther, *Zs. f. Ap.*, **7**, 106, 1933, have measured additional Balmer lines on objective-prism spectrograms.

[13] A description of the standardizing arrangement is given by E. G. Williams, *Mt. W. Contr.*, No. 487; *Ap. J.*, **79**, 280, 1934.

[14] For observational details see *Mt. W. Contr.*, No. 486; *Ap. J.*, **79**, 183, 1934.

which photometric tracings were made, are indicated in Table IV. Typical tracings are reproduced in Figures 2 and 3. The measurements are discussed under the following headings: "Structure of Lines," "Central Intensities," "Problem of Overlapping Wings," "Total Line Absorptions," and "Intensities of Emission Lines."

TABLE IV

SPECTROGRAMS USED FOR PHOTOMETRIC MEASUREMENTS

Star	Type	Balmer Series	Paschen Series
α Leonis	B8n	V316, 338	G744
α Lyrae	Ao	343, 380	757
α Canis Majoris	Ao	334, 494	737
β Orionis	cB8	324	742
α Cygni	cA2	336, 343, 380	776, 779
P Cygni	Beq	337, 358	581
γ Cassiopeiae	Boe	444	578

STRUCTURE OF LINES

The shapes of the lines have been determined in the usual manner by measuring from the photometric curves the widths corresponding to a succession of intensities ranging from that of the adjacent continuous spectrum to that at the center of the line. This method, which assumes the lines to be symmetrical, yields mean contours, the two sides of each line not being measured independently. The results are plotted in Figure 4 in terms of per cent absorption, PA, defined as

$$\frac{(PA)_\lambda}{100} = \frac{I_\lambda^c - I_\lambda}{I_\lambda^c},$$

where I_λ is the observed intensity at wave-length λ, and I_λ^c is the intensity which the continuous spectrum would have if the line were absent. I_λ^c is found by drawing on the tracing a smooth curve connecting the apparent positions of the continuous spectrum on either side of the line. In our observations relatively little uncertainty arises in locating the *apparent* continuous background, but for the crowded lines near the head of the Balmer series, and to a less degree in the Paschen series, the difference between the apparent and the

original undisturbed background may be considerable. This difficulty is considered at length in a later section.

If, as in Figure 4, a logarithmic scale is chosen for per cent absorption, most of the points for many of the hydrogen lines lie on a straight line, indicating that a large part of the contour may be represented by the exponential formula

$$(PA)_\lambda = (PA)_{\lambda_0} e^{-k|\lambda-\lambda_0|},$$

where λ_0 is the wave-length of the line center and k a constant. The outer limits of the lines, i.e., the widths where a drop below the continuous spectrum can first be detected, are arbitrarily plotted against a per cent absorption of 1. The points so fixed usually yield a reasonable continuation of the curve through the other points.

The centers of most of the lines exhibit less absorption than that called for by a continuation of the exponential form of the wings. In α Leonis the effect amounts to a decided flattening of the centers of the lines. In α Lyrae, on the other hand, the lines, while very wide, have well-marked cores. In the Balmer lines of α Cygni and β Orionis the exponential form continues practically to the center. A slight departure at the very center must be expected, of course, because the finite purity of the spectrograms would round off the sharp central cusp given by the formula.[15]

For the narrow-line stars β Orionis and α Cygni, the plotted points define nearly straight lines whose slopes for the first few members are essentially constant and then slowly increase in advancing from line to line toward the heads of the series (Fig. 4). In the broad-line stars α Leonis and α Lyrae, on the other hand, considerable curvature appears in many lines, although for one or two lines of smallest quantum numbers the contours are practically straight. In advancing down the series the wings apparently become less and less extensive, as evidenced by the increasing curvature exhibited by the plotted points—an effect, doubtless, of overlapping. The necessity of distinguishing between true and apparent shapes will be emphasized in the section on overlapping.

[15] $d(PA)/d\lambda \neq 0$ at the line center.

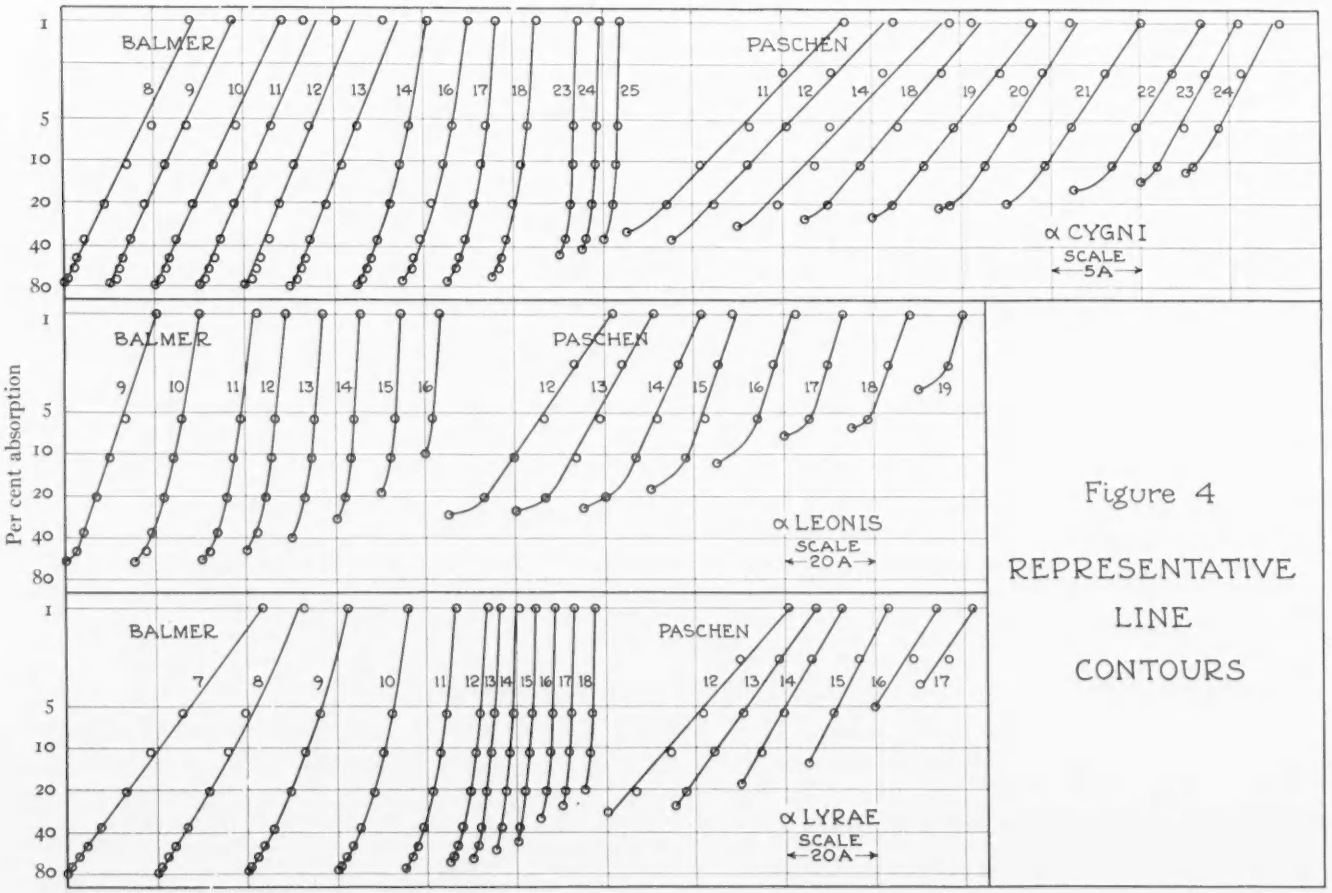


Figure 4

REPRESENTATIVE LINE CONTOURS

The contours of H9 λ 3835 in the spectra of α Cygni, α Lyrae, and α Leonis are compared on the same scale in Figure 5.

Mean contours from the Paschen series in several stars are compared with those from the Balmer series in Figure 6. The earliest available members of the Paschen series have been chosen because they are most intense and have the best determined contours. In the narrow-line stars, Balmer lines having the same or slightly smaller m's have been used; in the broad-line stars, to avoid effects of overlapping, those with m's about 4 units smaller than the Paschen line

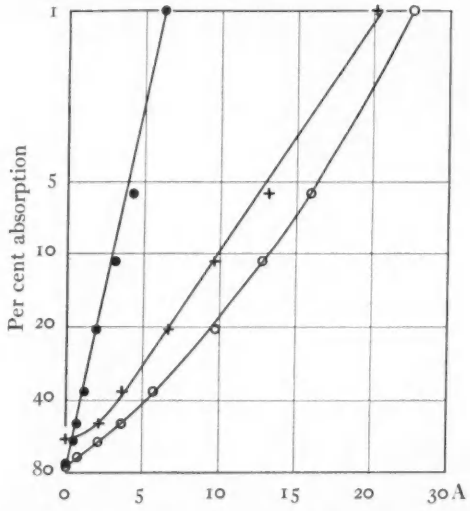


FIG. 5.—Contours of H9 (λ 3835). α Lyrae, circles; α Cygni, dots; α Leonis, crosses

have been used. The individual lines used for each star are noted in the figure. The full lines represent the original mean contours; the dashed lines the Balmer contours with ordinates reduced by a factor which gives the curve the same central intensity as that of the Paschen contour. The circles are located by multiplying the abscissae of the reduced curve by a factor, about 2.3, which is the ratio of the wave-lengths of the Paschen and Balmer lines involved. The circles, therefore, represent the hypothetical contours which the Balmer lines would have at the wave-lengths of the Paschen lines if their structure were on a pattern proportional to wave-length, and the central intensity reduced to that of the Paschen lines. It is interest-

ing to find that for the *c* stars β Orionis and α Cygni the circles lie remarkably near to the actual Paschen contours; and that for α Leonis, in whose spectrum the lines are extremely wide and diffuse, the correspondence of the circles to the Paschen contour is almost as good. The widths of the hydrogen lines in these three stars might, therefore, as far as the above-mentioned facts are concerned, be due to radial motion. The hypothesis of a quasi-solid rotation scarcely seems applicable to β Orionis and α Cygni, for in the spectra of these

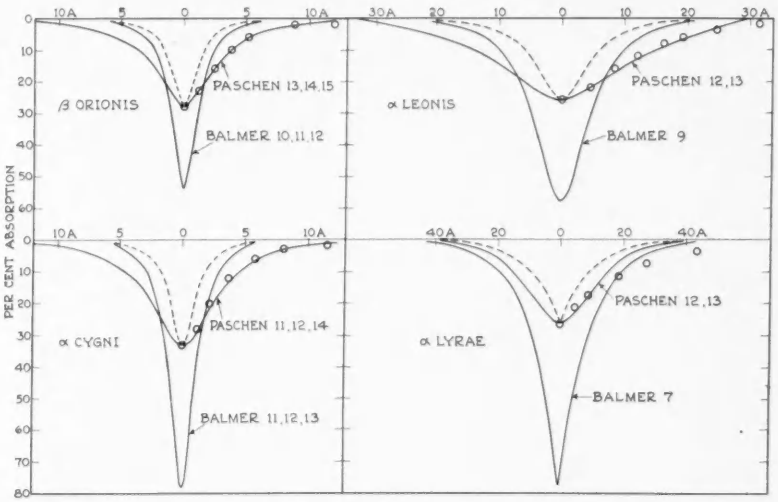


FIG. 6.—Comparison of mean contours of Balmer and Paschen lines

Solid lines, observed contours.
Dashed lines, Balmer contours with reduced ordinates.
Circles, reduced Balmer contours with ordinates multiplied by 2.3 ($=\lambda_P/\lambda_B$).

stars the lines of elements other than hydrogen are narrow with sharply defined edges. These stars may have very extensive hydrogen atmospheres with motions not shared by other atoms, or, the widths of the hydrogen lines may be ascribed to Stark effect. In α Leonis, since lines of all elements are wide, it can only be said that if the widening is due wholly to rotation it would involve a velocity of the order of 1500 km/sec., for whose control a very large mass would be required.

In α Lyrae the circumstances are quite different, for the circles representing the reduced Balmer contours (Fig. 6) do not at all fit

the Paschen contour but indicate considerably more intense wings. The same behavior is exhibited by α Canis Majoris (not included in Fig. 6). The widened lines in these stars are thus apparently not caused by radial motion but by some effect in which the displacements are not proportional to wave-length, possibly because the action on the Balmer series differs from that on the Paschen series.

The foregoing results bring out pronounced differences in the behavior of the hydrogen lines in early-type spectra classified as *n*, *s*, and *c*. The *s* stars have been regarded as intermediate between those of groups *n* and *c*, but our observations suggest that this relationship

TABLE V

WIDTHS OF PASCHEN EMISSION LINES IN γ CASSIOPEIAE

m	MEAN λ	MEASURED	CALCULATED		
			Equation	$H\alpha$, 8.7 A	$H\gamma$, 4.4 A
12–17	8590 A	9.5±0.5 A	12.4 A	11.4 A	8.7 A
19–22	8380	8.5±0.2	12 0	11.1	8.5

requires further consideration, at least in so far as stars typified by α Lyrae and α Canis Majoris[16] are concerned.

Our observations include two stars, γ Cassiopeiae and P Cygni, in whose spectra numerous lines of both hydrogen series are bright. A general comparison of the Paschen lines with those of the Balmer series can be made, although it is not possible to study their structure in great detail. Data for the widths of the Paschen lines in γ Cassiopeiae (see Pl. I) are summarized in Table V. For comparison with these values we may use R. H. Curtiss' measurements of Balmer lines [17] with additional observations of *Hα* by Otto Struve[18] and P. W. Merrill.[19]

Curtiss[17] has given the following equation for line widths, based on his measures of lines from *Hζ* to *Hα*:

$$\Delta\lambda = 0.001885(\lambda - 2020) .$$

[16] In these stars the hydrogen lines have fairly sharp cores bordered by extensive wings.

[17] *Pub. Obs. Univ. of Mich.*, **2**, 24, 1916.

[18] *Ap. J.*, **77**, 66, 1933. [19] *Lick Obs. Bull.*, **7**, 164, 1913.

As shown in Table V, widths for Paschen lines calculated from this equation are larger than the observed values. This discrepancy and the fact that according to the equation the line widths are proportional to $(\lambda - 2020)$ rather than to λ may arise from the effect on the measured widths of the increasing intensity of the hydrogen lines from $H\zeta$ to $H\alpha$, the stronger lines perhaps yielding too great relative widths. If this hypothesis is correct, we should expect widths for the Paschen lines calculated from the very intense $H\alpha$ line, on the assumption that width is proportional to wave-length, to lie between the measured values and those derived from the equation. The Paschen widths calculated from a width of 8.7 A for $H\alpha$, shown in the fifth column of Table V, fulfil the expected relationship. The intensity of bright $H\gamma$, on the other hand, is comparable to that of the Paschen lines measured; and the Paschen widths calculated from this line, last column of Table V, agree, probably within errors of measurement, with the observed values. Hence, as far as may be judged from the present none too precise data, the widths of comparable emission lines in the two hydrogen series in the spectrum of γ Cassiopeiae appear to be proportional to wave-length. This would be true, of course, if the widths were caused by radial motion.

In the spectrum of P Cygni, as is well known, the Balmer series consists of strong bright lines each having a dark companion on the short wave-length side. Our infra-red spectrogram of this star, G581, taken on July 19, 1932, while of poor quality, shows clearly that the structure of the Paschen lines is qualitatively similar to that of the Balmer lines. It is not possible to make satisfactory measurements of line widths, but the distance between the effective centers of the bright and dark components, $E-A$, can be determined with a fair degree of accuracy. Micrometer measures of eight lines, $m=12$ to $m=20$ (omitting $m=18$), mean wave-length 8540 A, yield $E-A=3.26\pm0.04$ A. Attempts to read this quantity from the photometric tracing led to a slightly higher value, about 3.46 A in the mean. To obtain a comparable value for the Balmer series, five lines, $m=9$ to $m=14$ (omitting $m=10$), mean wave-length 3760 A, were measured on plate V10, taken June 22, 1931, with the result $E-A=1.47\pm0.02$ A. Multiplying the Balmer value by the ratio of the mean wave-lengths in the two series gives 3.34 A. Hence the

separations of the bright and dark components in the two series are probably proportional to wave-length within errors of measurement.

Discussion of the total intensities of the bright lines in the spectra of γ Cassiopeiae and P Cygni is deferred to the last section.

CENTRAL INTENSITIES

Photometric measurements of the per cent absorption at the centers of numerous hydrogen lines in several stars are collected in Table VI. These are *apparent* central absorptions determined as

TABLE VI

APPARENT CENTRAL ABSORPTION

(Per Cent)

m	α Cygni cA2		β Orionis cB8		α Leonis B8n		α Can. Maj. Ao		α Lyrae Ao	
	Bal.	Pas.	Bal.	Pas.	Bal.	Pas.	Bal.	Pas.	Bal.	Pas.
6	.....	.....	.....	.....	.....	.....	80	.....	.....	.....
7	.....	.....	.....	.....	.....	.....	80	.....	79	.....
8	76	.....	52	.....	.....	.....	75	.....	78	.....
9	76	.....	52	.....	58	.....	75	.....	77	.....
10	78	.....	52	.....	60	.....	73	.....	76	.....
11	78	32	54	.....	58	.....	70	.....	73	.....
12	78	37	56	32	49	28	64	28	66	28
13	80	.....	54	30	40	26	60	22	62	26
14	78	30	53	26	30	24	50	14	54	18
15	76	.....	50	26	19	18	40	12	46	13
16	75	.....	53	22	10	12	28	8	34	4
17	75	.....	51	21	.....	8	21	4	26	3
18	69	26	44	.....	.....	6	8	.....	20	.....
19	68	25	36	15	.....	3	.....	.....	11	.....
20	63	22	31	13	.....	.....	.....	.....	.....	.....
21	60	20	22	12	.....	.....	.....	.....	.....	.....
22	53	16	.....	.....	.....	.....	.....	.....	.....	.....
23	47	14	.....	.....	.....	.....	.....	.....	.....	.....
24	43	12	.....	.....	.....	.....	.....	.....	.....	.....
25	37	.....	.....	.....	.....	.....	.....	.....	.....	.....

described on page 27, and in some instances may be less than the "true" absorptions which would be found if each line could be observed separately. The mutual interaction of neighboring lines will be discussed in the next section.

Certain features of the apparent central intensities are exhibited by Figure 7, in which are plotted the data of Table VI together with a few values for early members of the Balmer series that are means

of determinations by C. T. Elvey[20] and E. G. Williams.[21] In general the Paschen lines in a given star are decidedly less intense at the center than the corresponding Balmer lines. An exception is offered, however, by α Leonis in whose spectrum the Balmer intensity drops below that of the Paschen lines at $m = 16$. This spectrum is the only one observed in which the ratio of Balmer intensity to Paschen intensity decreases from $m = 12$ to $m = 16$; in all others there is a well-marked *increase* in the ratio. The relatively slow decrement of the Paschen series in α Leonis is illustrated by the fact that this series can be observed to $m = 19$, the Balmer series only to $m = 16$ or

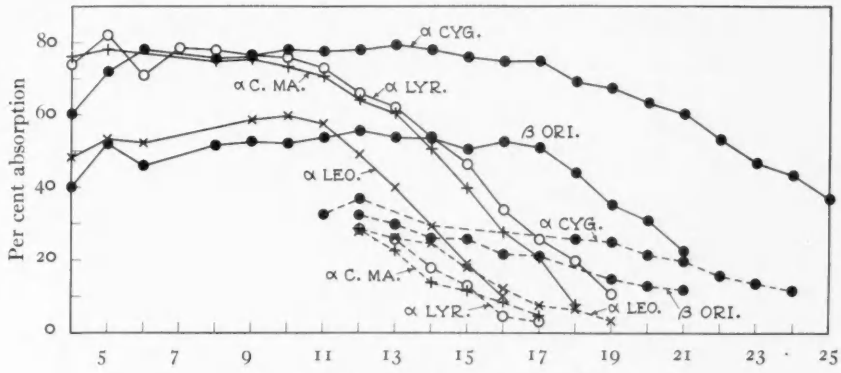


FIG. 7.—Apparent central absorptions
Solid lines, Balmer series.
Dashed lines, Paschen series.

17 (Table III). Because the Balmer lines are nearer together and more intense than the corresponding lines of the Paschen series, overlapping of the wide, diffuse lines would tend to produce the observed effect. The Paschen series as a whole appears relatively intense in α Leonis; for example, all observed members are as strong as, or stronger than, the corresponding lines in α Lyrae and α Canis Majoris, while the Balmer series throughout the whole observed range from $m = 4$ to $m = 16$ is decidedly weaker. The higher temperature of the atmosphere of α Leonis would give a larger Boltzmann factor and hence a relatively greater number of absorbing atoms to the third quantum level from which the Paschen lines arise.

[20] *Ap. J.*, **71**, 191, 1930.

[21] *Annals of the Solar Physics Observatory, Cambridge*, **2**, 25, 1932.

The relatively slow decrements and the resulting extensions toward short wave-lengths of both series in the narrow-line stars β Orionis and α Cygni are well illustrated by Figure 7. This behavior, which is ascribed to the comparative absence of overlapping, is apparently characteristic of *c* stars;[22] the cause probably lies in the low density of the hydrogen atmosphere.

PROBLEM OF OVERLAPPING LINES

In comparing the two hydrogen series it is desirable to use lines having the same or adjacent upper levels, and with the present observations this procedure leads at once to the problem of overlapping wings. In the Paschen series P11 has been observed a few times, but in general P12 is the first line from which useful results may be obtained.[23] In this series the lines appear not to interfere with each other until P14 or 15 is reached (perhaps P19 or 20 for the narrow-line stars), but in the Balmer series, with its closer spacing and more intense lines, overlapping of wings of successive lines appears to begin at H8 in α Lyrae and α Canis Majoris, at H9 or 10 in α Leonis, and at H14 or 15 in β Orionis and α Cygni. The beginning of interference of any line with the following line may be judged in two ways: first, by noting on the photometric tracings (Figs. 2 and 3) the first interval between successive lines in which there appears to be no straight-line portion (undisturbed continuous background); and, second, by assuming the beginning of overlapping to be marked by deviation of an individual contour from the exponential form exhibited by undisturbed lines (see Fig. 4 and p. 28). The fact that these effects become apparent at about the same point in the series supports the hypothesis that the wing of one line may exert absorption at points within another line. Farther along the series, the overlapping appears, on reasonable assumptions, to be so extensive that five or more lines may contribute to the absorption at a particular wave-length.

Theoretically, the original contours might be derived from the

[22] In γ Cygni, cF8, the Paschen lines are strong and exhibit an extremely small decrement.

[23] Very recent improvements in infra-red plates should make the observation of P11, and possibly of P7, a matter of no great difficulty. The lines P8, 9, and 10 lie in a region where atmospheric lines interfere.

apparent ones by the following procedure: Because of the decreasing intervals between the members of the series, it is obvious that as the lines approach each other there will be one in which the short wave-length wing alone is affected by another line, the wing on the long wave-length side remaining undisturbed. The difference between the two wings of this line may be ascribed to the additional absorption of the long wave-length wing of the next line, whose undisturbed form may then be determined; and so on down the series. We have not considered the present data sufficiently accurate to yield a reliable result in this way. We have attempted, however, to gain some idea of the original structure of the series in the spectrum of α Lyrae by making various reasonable assumptions concerning it and then computing the effects of overlapping in the hope eventually of reproducing the observed photometric data. The computation, once a particular assumption has been made, is straightforward, although laborious, and can be carried through with any desired degree of accuracy.

It was noted in the section on the structure of lines that in those lines which are free from overlapping the slopes of the exponential contours are nearly constant. This conclusion is borne out by measurements of the $H\beta$ and $H\gamma$ lines by C. T. Elvey.[24] All the Balmer lines were therefore assumed to have the same slope as that measured for $H\epsilon$ (see Fig. 4). Calculations were then based on several configurations of the hypothetical central intensities, each curve having a smooth maximum near H11, and beyond H13 a uniform decrement, less rapid than that observed. The final set of points is shown in Figure 8.

The actual computations employed a graphical method, based on the validity of the equation $I = I_0 e^{-\Sigma \tau_m}$, where τ_m is the optical thickness of the stellar atmosphere for the line having quantum number m. On semi-logarithmic co-ordinate paper were plotted a series of straight lines (exponential contours), all having the same slope but with intercepts on the vertical axis equal to the central absorptions assumed for successive lines. Then by so placing a separate strip of paper on which the positions of the Balmer lines were plotted that any given line lay on the axis of the first plot, the absorption due to

[24] *Loc. cit.*

that line at the centers of other lines and at intermediate points could be read off. In this way the effect of each line at the center of every other line and at points midway between the lines was determined. The separate values of the per cent absorption were then converted into values of $\Delta \log I$ by means of the equation

$$\Delta \log I = \log \frac{I_0}{I} = -\log \left(1 - \frac{PA}{100}\right).$$

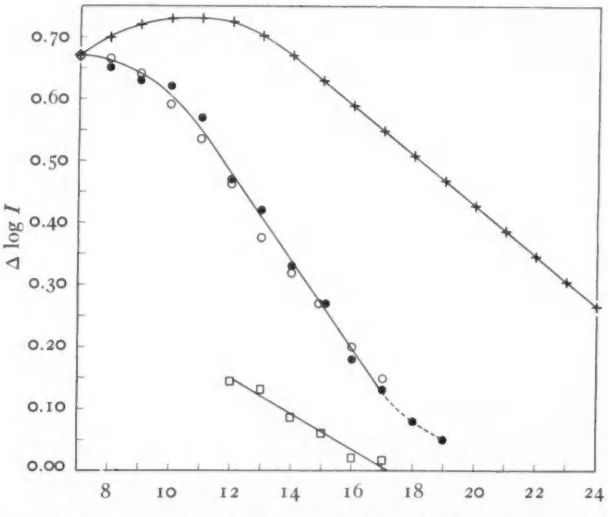


FIG. 8.—Observed and calculated central intensities in the spectrum of α Lyrae

Dots, observed intensities in the Balmer series.
Squares, observed intensities in the Paschen series.
Crosses, assumed true intensities in the Balmer series.
Circles, calculated intensities in the Balmer series.

After summing all the values of $\Delta \log I$ which belong together, the central intensities were formed by subtracting the values for each line center from the mean values for the background on either side. These differences are found to correspond fairly well to the observed central intensities (Fig. 8).[25] From the standpoint of our special assumptions, therefore, we may take the assumed curve as represent-

[25] A given discrepancy in per cent absorption (especially among the larger values) is greatly magnified by the $\Delta \log I$ scale adopted in Fig. 8. A still closer correspondence could, of course, be obtained by modifying the assumed central intensities, but in the present status of the problem more precise results are of little importance.

ing the "true" central intensities. It is of interest to note that the Balmer decrement derived from the true curve agrees approximately with the observed decrement in the Paschen series where overlapping should be relatively ineffective.

The additive process outlined above, slightly modified in its numerical application, has been used to calculate the contours of H8, H9, and H13 from precisely the same assumptions as were made for the computation of central intensities, namely, that the true central intensities are those shown in Figure 8, and that wings of all the lines have the same slope as that measured for $H\epsilon$. The computed

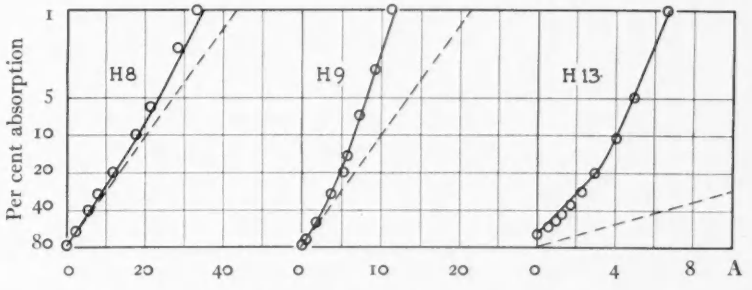


FIG. 9.—Observed and calculated contours of H8, H9, and H13 in the spectrum of α Lyrae.

Circles, observed points.
Dashed line, assumed original contour.
Solid line, calculated contour.

values agree surprisingly well with the observed contours, even for H13, where the modification of the original contour is very great (see Fig. 9). At first glance the agreement appears to be strong evidence that the optical phenomena in the atmosphere of α Lyrae correspond to the additive process in our calculation of overlapping, but certain considerations tend to weaken this conclusion. In the first place, the agreement of calculated and observed points at the background, $PA = 1$, necessarily follows from the condition of overlapping lines on either side. In other words, the apparent width of a line must come out very nearly half the distance between the preceding and the following lines. Moreover, subject to certain strongly restrictive physical conditions, the assumed true central intensities have been adjusted to yield the observed values. For example,

even if the widths assumed for the true wings be reduced by one-half (and the assumed central intensities be readjusted), we still arrive at a reasonable approximation[26] to the observed contour.

We now turn to a difficulty of another kind which may be of considerable general interest. According to the assumptions underlying the computation described above, the continuous spectrum at λ 4035 A, midway between H6 and H7, suffers no absorption from hydrogen lines, while at λ 3694 A, midway between H17 and H18, the calculations show that the overlapping of wings of numerous Balmer lines reduces the intensity to one-twentieth that of the original photospheric spectrum. This reduction appears to be far greater than that exhibited by the actual spectra.[27] The predicament is that our hypothesis, while reproducing satisfactorily the observed line structure, gives entirely too much general absorption all along that part of the spectrum where the lines are closely packed. The difficulty is not limited to our particular numerical assumption, but apparently will occur in any hypothesis which attempts to represent the observed lines by assuming the original lines all to have the same form with gradually diminishing intensities.[28]

The difficulty might be avoided by assuming that in passing down the series the wings diminish in intensity more rapidly than the cores, but we cannot adduce reasons why the lines should behave in this manner. In fact, if the wings are produced by Stark effect, their intensities relative to the cores should increase toward the head of the series.

Another possibility is that the energy trapped by the overlapping lines, instead of reacting to raise slightly the effective temperature of the general stellar radiation, is re-emitted as continuous spec-

[26] The curve so obtained, however, does not represent the observations quite as well as that based on the full width. Future observations with higher dispersion and greater photometric accuracy should greatly reduce the allowable uncertainty in the assumptions.

[27] C. S. Yü's measurements (*L.O.B.*, **12**, 155, 1926) indicate about one-half as much energy at λ 3694 A as at λ 4035 A.

[28] A few years ago, before the great extent of the line wings was fully realized, the current difficulty was to explain why the intensity of the continuous spectrum between the lines is as *low* as it is; now we are troubled because the intensity is not lower.

trum only in the region of the lines themselves. This explanation raises a rather general physical problem which may be related to that of the continuous absorption (or emission) beyond the head of the series.[29]

The computations concerning overlapping are based on the equation $I = I_0 e^{-\Sigma\tau}$, which corresponds to pure absorption (no scattering). For small values of $\Sigma\tau$, $I = I_0/(1+\Sigma\tau)$, the equation for scattering,[30] gives nearly the same value of I_0/I as the exponential equation, but for the relatively large value of $\Sigma\tau$ arising in the assumption of overlapping hydrogen lines in α Lyrae it gives, instead of 20, about 5, a value in less violent disagreement with observation. Perhaps the anomaly discussed above arises because scattering cannot properly be neglected.

TOTAL LINE ABSORPTION

Measurements of the total absorption in hydrogen lines in the spectra of α Cygni, β Orionis, and α Leonis are given in Table VII. These are apparent values as measured from the photometric tracings in the usual way, without correction for overlapping.

Unsöld[31] has shown that the number of hydrogen atoms in the second quantum level may be calculated from the total absorption of a Balmer line. For the higher members of the series, where re-emission of the absorbed light is neglected, he used the equation

$$\int \tau d\lambda = \frac{\pi e^2 \lambda^2}{mc^2} fN ,$$

[29] Reliable data on the intensity of the continuous spectrum just beyond the head of the Paschen series would be extremely interesting. Unfortunately, determinations are made difficult by the fact that the head of the series lies in a rather dense group of atmospheric lines (see Fig. 1). Our spectrograms are unsuited to such determinations and no measurements have been attempted. Visual comparison of the plates suggests that in the emission-line star γ Cassiopeiae the continuous spectrum beyond the head of the series is relatively more intense than in the absorption-line stars, but the effect appears not to be very pronounced.

[30] A. Unsöld, *Zs. f. Phys.*, **59**, 353, 1929.

[31] *Ibid.*

where N is the number of atoms and f the so-called oscillator strength. τ is the optical depth; it is assumed that

$$I = I_0 e^{-\tau} .$$

TABLE VII

TOTAL LINE ABSORPTIONS, $\int \log_e \frac{I}{I_0} d\lambda$, AND NUMBERS OF ATOMS IN SECOND AND THIRD LEVELS

m	α Cygni				β Orionis				α Leonis			
	Balmer		Paschen		Balmer		Paschen		Balmer		Paschen	
	Abs.	log N_2	Abs.	log N_3	Abs.	log N_2	Abs.	log N_3	Abs.	log N_2	Abs.	log N_3
6	2.74	14.93	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
7	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
8	2.96	15.34	.....	.....	1.90	15.25	.....	.....	.....	.....	.....	.....
9	3.59	15.71	.....	.....	1.99	15.45	.....	.....	9.26	16.12	.....	.....
10	3.84	15.90	.....	.....	2.10	15.62	.....	.....	8.77	16.26	.....	.....
11	3.41	15.99	2.76	14.98	2.05	15.77	.....	.....	7.19	16.32	.....	.....
12	3.47	16.12	3.00	15.13	2.11	15.90	2.25	15.00	4.46	16.23	8.60	15.58
13	3.48	16.22	.....	.....	1.98	15.98	2.54	15.16	2.84	16.13	6.95	15.60
14	2.95	16.24	2.26	15.22	1.82	16.03	2.02	15.18	1.62	15.98	6.16	15.66
15	.....	.....	.....	.....	1.69	16.12	2.09	15.28	0.84	15.82	3.52	15.51
16	2.65	16.39	.....	.....	1.89	16.25	1.78	15.30	0.34	15.50	1.96	15.34
17	2.41	16.44	.....	.....	1.40	16.21	1.59	15.32	.....	.....	1.17	15.19
18	1.92	16.41	1.91	15.50	1.14	16.19	.....	.....	.....	.....	0.94	15.19
19	.....	.....	1.75	15.53	.....	.....	0.96	15.27	.....	.....	0.45	14.94
20	.....	.....	1.40	15.51	.....	.....	0.77	15.25	.....	.....	.....	.....
21	.....	.....	1.08	15.45	.....	.....	0.76	15.30	.....	.....	.....	.....
22	.....	.....	1.00	15.49	.....	.....	.....	.....	.....	.....	.....	.....
23	0.68	16.29	0.64	15.35	.....	.....	.....	.....	.....	.....	.....	.....
24	0.54	16.24	0.50	15.30	.....	.....	.....	.....	.....	.....	.....	.....
25	0.46	16.23	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

To apply Unsöld's equation to our data, we write it in the form

$$\frac{\pi e^2 \lambda^2}{mc^2} fN = \int \Delta \log_e I d\lambda = 2.30 \int \Delta \log_{10} I d\lambda .$$

The integral on the right corresponds to the measured total intensity. Values of f are included in Table I; the first nine for each series are taken from Sugiura,[32] while the remainder were calculated from

[32] *Scientific Papers of the Inst. of Phys. and Chem. Research*, Tokio, **11**, 1, 1929.

the asymptotic formula $f = \text{const.}/m^3$. The values of log N_2 and log N_3 in Table VII, computed from Balmer and Paschen lines respectively, are plotted in Figure 10. If the theory were correct and the measures accurate, all values of N_2 for a particular star should be equal, as should also be those of N_3. As a matter of fact, the values from successive lines rise to a maximum and then decrease, slightly in the narrow-line stars α Cygni and β Orionis, very decidedly in the broad-line star α Leonis. The probable interpretation is that re-emission weakens the earlier lines in each series, thus invalidating the theory, while the observed intensities of the later lines are

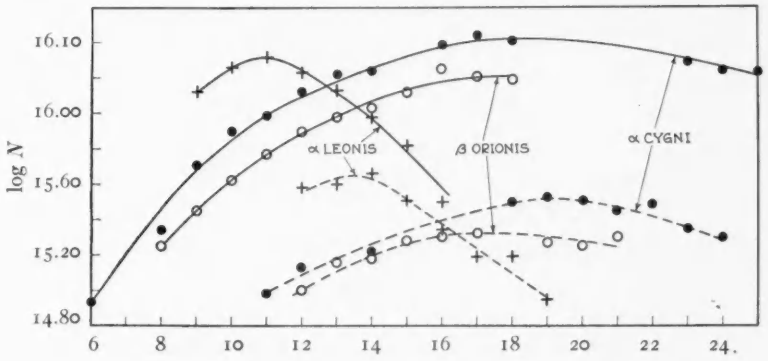


FIG. 10.—Numbers of atoms in second and third energy states
Solid lines, Balmer series.
Dotted lines, Paschen series.

reduced by overlapping. Our largest values of N_2 and N_3, as read from the curves in Figure 10 (listed in Table VIII), will thus be equal to or less than the true values.

On the assumption that the hydrogen atmosphere is in thermal equilibrium, its temperature may be computed from the relative number of atoms in the second and third levels by the Boltzmann equation

$$N_3 = N_2 \frac{g_3}{g_2} e^{-\frac{E_3 - E_2}{kT}},$$

which for the present case becomes

$$\frac{9400}{T_0} = \log \frac{N_2}{N_3} + 0.35 .$$

The temperatures computed from this equation are in the last column but one of Table VIII. As they represent average temperatures throughout the strata occupied by absorbing hydrogen atoms, they are equal to or less than the so-called boundary temperature, T_0. The effective photospheric temperature, T_e, is found by multiplying T_0 by the fourth root of 2 (last column of Table VIII). The results

TABLE VIII

NUMBERS OF HYDROGEN ATOMS IN THE SECOND AND THIRD LEVELS

Star	log N_2	log N_3	log N_2/N_3	T_0	$T_e = \sqrt[4]{2}\, T_0$
α Cygni	16.42	15.50	0.92	7,400°	8,800°
β Orionis	16.21	15.32	.89	7,600	9,000
α Leonis	16.30	15.62	0.68	9,100	11,000

for α Cygni and β Orionis have much more claim to confidence than that for α Leonis, where the value of log N_2/N_3 may be considerably affected by overlapping. In α Lyrae and α Canis Majoris overlapping is so great that computation of temperatures by this method would seem to be almost meaningless.

INTENSITIES OF EMISSION LINES

The presence of bright lines in the Balmer series of γ Cassiopeiae and P Cygni is well known. Our observations show that numerous members of the Paschen series also are bright and that, at least to a first approximation, their structure is related to that of corresponding Balmer lines by the equation $\Delta\lambda = c\lambda$ (see p. 33).

Photometric measurements of the central (or maximum) intensity of lines of both series are shown in Figure 11. The quantities plotted are $\Delta \log I = \log [(I_c + I)/I_c]$, where I_c stands for the intensity of the neighboring continuous spectrum. The results for the Paschen series in γ Cassiopeiae deserve special mention. It is not certain that the maximum in the curve of apparent intensities represents the actual behavior of the lines as emitted by the stellar atmosphere. In the first place, the measured intensities of the last three or four lines may be lowered because the observed minima between the lines do not extend to the true level of the continuous spectrum (i.e., the bright lines overlap). Moreover, it is probable that the measured

intensities of lines, $m = 13$, 15, and 16, are increased by blending with emission lines of *Ca* II.[33] When these facts are taken into consideration, the evidence for an anomalous maximum near $m = 16$ is not very strong.

A knowledge of the relative intensities of pairs of Balmer and Paschen lines which arise by transitions from the same upper levels

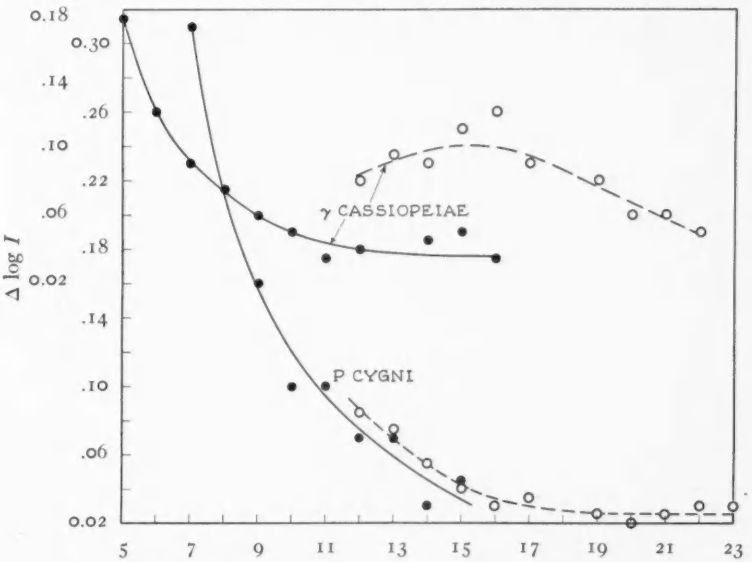


FIG. 11.—Observed intensities of emission lines in γ Cassiopeiae and P Cygni. Scale at left refers to γ Cassiopeiae.

Dots, Balmer series.
Circles, Paschen series.

allows certain conclusions concerning physical conditions in the hydrogen atmosphere and the effective temperature of the photosphere. An outline of the underlying theory follows.

Let π_{mn} be the probability that an atom in quantum state m will, in unit time, perform the transition $m \to n$ and emit radiation ν_{mn}. Then from a constant number, N_m, of atoms in level m, the steady emission per unit time will be

$$E_{mn} = h\nu_{mn} N_m \pi_{mn} .$$

[33] The influence of the calcium lines is shown also by the negative displacements measured for the three blended Paschen lines. Emission at the related calcium lines H and K, if present, is of extremely low intensity.

Now assume that the atmosphere of emitting hydrogen about the star is transparent to its own radiation and that it is sufficiently tenuous to make negligible the effects of collisions. Then the relative intensity of Balmer and Paschen lines from the upper level m, emitted by a given homogeneous volume, would be

$$\frac{E_{m2}}{E_{m3}}=\frac{\nu_{m2}}{\nu_{m3}}\frac{\pi_{m2}}{\pi_{m3}},$$

if we could disregard the division of levels 2 and 3 into sublevels having different azimuthal quantum numbers. The result, however, is actually dependent on the relative numbers of atoms in these various sublevels, and we must write the more complete expression

$$\frac{E_{m2}}{E_{m3}}=\frac{\nu_{m2}}{\nu_{m3}}\left[\frac{N_{mS}\pi_{mS\to 2P}+N_{mP}\pi_{mP\to 2S}+N_{mD}\pi_{mD\to 2P}}{N_{mS}\pi_{mS\to 3P}+N_{mP}(\pi_{mP\to 3S}+\pi_{mP\to 3D})+N_{mD}\pi_{mD\to 3P}+N_{mF}\pi_{mF\to 3D}}\right]$$

or

$$\frac{E_{m2}}{E_{m3}}=f(N,\pi)\,. \qquad (a)$$

The numerical value of the expression in the bracket depends not only upon the relative numbers of atoms in the various sublevels, but to some extent also upon the density of radiation streaming through the gas, for the radiation determines the amount of induced emission. In most laboratory experiments the induced emission is inappreciable, but calculation shows that in considering the atmospheres of B-type stars it cannot be entirely disregarded. Some idea of its effect upon the ratio E_{m2}/E_{m3} may be obtained in the following way:

$$\pi_{mn}=A_{mn}+u_{mn}B_{mn}\,,$$

where A and B are respectively the Einstein coefficients of spontaneous and induced emission and u_{mn} is the density of radiation of frequency corresponding to the transition $m\to n$. If the radiation density is that of a *hohlraum* at temperature T, we may, since

$$B_{mn}=\frac{c^3}{8\pi h\nu^3}A_{mn}\,,$$

write the expression for π in the form

$$\pi_{mn} = A_{mn}\left(1 + \frac{1}{e^{h\nu/kT} - 1}\right).$$

The value of $h\nu/k$ for the $H\alpha$ line, for example, is 21,900; and it is obvious that for temperatures of 15,000° or higher the amount of induced emission relative to that of spontaneous emission may be considerable.

If within the bracket of equation (*a*) we replace π by A, thus omitting uB, we may, to compensate the omission, write outside the bracket the parameter Γ. Thus

$$\frac{E_{m2}}{E_{m3}} = \Gamma \times f(N, A). \qquad (b)$$

Γ, in a stellar atmosphere of emitting hydrogen, depends on the temperature of the photosphere and the distance above it. Table IX gives, for three temperatures, values of Γ for H14 and P14 for thin spherical shells at various distances from the center of the star. The unit of distance is the radius of the photosphere. The proper value of Γ for integrated light as observed from the earth cannot be de-

TABLE IX

NUMERICAL VALUES OF Γ

Dist. from Center	Photospheric Temperature		
	15,000°	25,000°	35,000°
1R	0.836	0.745	0.692
2	0.972	0.950	0.935
5	1.000	1.000	0.990
10	1.000	1.000	1.000

termined without more knowledge than we possess of the structure of the stellar atmosphere. If the atmosphere is extensive, Γ is nearly unity. To calculate the bracket in equation (*b*) we take $\Gamma = 0.9$; it seems unlikely that this can be sufficiently in error to invalidate the result. The A's are calculated from the equation

$$A_{mn} = \frac{3}{\tau}\frac{g_n}{g_m} f_{mn},$$

where $\tau = 4.51\ \lambda^2$. The f's are those for individual transitions from one sublevel to another and are found by a simple algebraic process from the total f's in Table II. Table X shows the ratios $E_{14,2}/E_{14,3}$ for the lines H14 and P14 calculated from equation (b) for various simple distributions of atoms according to azimuthal quantum number. Line e corresponds to the distribution in thermal equilibrium.

Application of these results to data for γ Cassiopeiae and P Cygni may now be made. For most of the lines the measured quantities are

TABLE X

CALCULATED RELATIVE INTENSITY OF LINES H14 AND P14

HYPOTHESIS	ASSUMED DISTRIBUTION IN AZIMUTHAL QUANTUM NUMBER				$E_{14,2}/E_{14,3}$
	N_S	N_P	N_D	N_F	
a	1	0	0	0	3.3
b	0	1	0	0	6.5
c	0	0	1	0	6.0
d	0	0	0	1	0.0
e	1	3	5	7	3.1

small and their percentage errors large. Uncertainty also arises from the fact that the violet and the infra-red plates of each star were taken about a year apart and it is possible that small changes may have taken place in the spectra. We assume the structure of lines in the two series to be similar, i.e., $\Delta\lambda/\lambda = \text{Const.}$ (see p. 33). Thus if I be the intensity of the emission at a particular point (say the maximum intensity) and E the total energy in the emission line, then

$$E = kI\Delta\lambda$$

and

$$\frac{E_B}{E_P} = \frac{I_B\lambda_B}{I_P\lambda_P}.$$

Let r be the ratio of intensity of emission to that of the neighboring continuous spectrum, and ρ the ratio of intensity of continuous spec-

trum near a Balmer line to that near the corresponding Paschen line. Then

$$\rho = \frac{E_B \, r_P \, \lambda_P}{E_P \, r_B \, \lambda_B}.$$

The photometric data are as tabulated. The values of ρ correspond-

Star	Balmer 14		Paschen 14	
	$\Delta \log I$	r_B	$\Delta \log I$	r_P
γ Cass	0.04	0.10	0.09	0.23
P Cygni	0.05	0.12	0.05[6]	0.14

ing to the five values of E_B/E_P in Table X, together with the corresponding black-body temperatures, are in Table XI. Impossible values of the temperature are omitted.[34] As hypothesis *a* (Table X) is physically improbable, the data for γ Cassiopeiae appear to support hypothesis *e*, i.e., distribution of atoms according to thermal equilibrium. Our results indicate, in accordance with several previous determinations, that γ Cassiopeiae is hotter than P Cygni; but the temperatures derived are higher than those usually found for these stars by direct observations of intensities in the continuous spectrum.[35] A discrepancy in this sense might be caused by space absorption, which tends to reduce the temperature derived directly from the continuous spectrum but has no effect on the results by our method.[36] The present results, while suggestive, especially for P Cygni,[37] are not sufficiently accurate to allow reliable conclusions;

[34] Case *d* is evidently impossible because emission lines are actually observed in both series; while for $\rho > 28.4$ there is no real positive value of T.

[35] The temperatures derived from our data by the method described would be lowered by assuming that more atoms are in the F-state than are called for by thermal equilibrium, but we are unaware of a physical justification for such an assumption.

[36] On the plausible assumption that emission lines and the neighboring continuous spectrum are equally absorbed in interstellar space.

[37] Comparison of our values of ρ with those corresponding to the temperatures found by usual methods, say 15,000° for γ Cassiopeiae and 7000° for P Cygni, gives differential space absorption at λ 3700 compared to λ 8600 of 0.4 mag. for γ Cassiopeiae, and either 2.0 mag. (hypothesis *b*) or 1.2 mag. (hypothesis *e*) for P Cygni.

but with better photometric data the method appears competent to yield valuable information concerning space absorption. Its application to abnormally colored Be stars would be particularly interesting.

TABLE XI

PHOTOSPHERIC TEMPERATURES DERIVED FROM INTENSITIES OF EMISSION LINES

HYPOTHESIS*	γ CASSIOPEIAE		P CYGNI	
	ρ	T	ρ	T
a	18	27,000°	8.6	12,000°
b	36	.....	16.9	24,000
c	33	.....	15.6	22,000
d	.....	.....	.....	.....
e	17	25,000	8.1	12,000

* See Table X.

The fact that the emission lines whose intensities are compared have a common upper level is a great advantage in the preceding discussion because it eliminates serious uncertainties and makes possible a relatively simple theoretical treatment.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
March 1934

ON THE SPECTRUM OF β LYRAE

By HELEN M. PILLANS

ABSTRACT

The wave-lengths of 240 absorption lines in the spectrum of the B8 component of β Lyrae are given. Of these, 176 are identified with *H*, *He* I, *Ca* II, *Si* II, *Si* III, *Cr* II, *Sc* II, *Ni* II, *Mg* II, *Fe* I, *Fe* II, *Ti* II, *Sr* II, *S* II, *N* II, and *V* II. A comparison of this component of β Lyrae with the supergiants β Orionis (B8 p) and σ Cygni (Aop) is made, with the result that the so-called B8 component is found to be normal for the later class of B9.

The lines in the spectrum of the B5 component are compared with those in the spectrum of 67 Ophiuchi (B5) and, as many important B5 lines are missing, the conclusion is reached that the B5 spectrum is not normal for its class.

A plate with phase $0^d.38$, which showed exceptionally strong *H* and *He* emission, showed also additional emission lines not seen at other phases. These are identified with *N* II 4621, *N* II 4630, *N* II 4643, and *Mg* II 4481.

During the years 1932 and 1933, forty-two spectrograms of β Lyrae were obtained with the Bruce spectrograph of the Yerkes Observatory. All of these plates were taken on the fine-grained Eastman Process emulsion and showed a number of features which had not been previously recorded.

I

The spectrograms mentioned above were taken with one prism and have a dispersion of 30 A/mm at λ 4500. An inspection of the plates showed that those taken on February 15.46, February 21.46, and March 1.43, 1933, were of the best quality, and they were consequently chosen for the measurement of wave-lengths. The phases were computed by F. Hellerich's formula:[1]

$$\text{Min} = 2398590.534 + 12.9080286E + 0.00000392 3E^2 - 0.000000000012E^3.$$

On two of the pates, the B8 and B5 component spectra are well separated, with the B8 lines displaced toward the red; on the third plate the lines are blended.

These three plates were measured and reduced and the B8 absorption lines were corrected for the radial velocity of the B8 component by means of nineteen well-identified, unblended metallic lines belonging to the B8 spectrum. Table I gives the measured wave-

[1] *Astronomische Nachrichten*, **216**, 37, 1922.

TABLE I

Star I.A.	No. of Plates	Intensity	Identification	Star I.A.	No. of Plates	Intensity	Identification
3923.30	1	1	*S* II 3.43(6)	4032.60	3	1n	*S* II 2.77(6)
3926.43	3	3	*He* I 6.53(1)	4035.25	3	1nn+	*V* II 5.62(40)?; *N* II 5.09(4n)
3931.23	2	2	*S* II 1.90(5)				
3933.59	3	5	*Ca* II(K) 3.66 (400 *R*) *S* II 3.25 (7)	4038.49	3	1	*Cr* II 8.04(2)
				4041.39	3	2	*N* II 1.32(5n)
				4043.96	3	1	*N* II 3.54(3n)
3935.91	3	2	*He* I 5.91(1)	4045.87	3	1	*Fe* I 5.83(60r)
3938.39	1	2	.....	4048.98	3	1n+	†
3940.40	2	1	.....	4051.77	3	1	*Cr* II 2.02(1)
3944.04	2	1	.....	4053.25	2	1	.....
3946.88	2	1	.....	4056.82	2#	1	.....
3951.63	2	1	*V* II 1.97(40)?; *C* II 2.08(1)	4058.92	2	1	.....
				4061.82	2	1	.....
3955.69	3	1	*N* II 5.85(6)	4063.42	3	1+	*Fe* I 3.61 (45)
3957.86	2	1	*Cr* II 8.09(1)	4067.04	3	3	*Ni* II 7.04(3)
3964.67	3	4	*He* I 4.73(4)	4069.90	1##	2n	*S* II 9.89(1); *S* II 9.65(1)
3966.14	2	2	.....				
3968.21	1	4	*Ca* II (H) 8.46 (350*R*)*	4072.82	3	1n+	*N* II 3.04(2n)
				4075.30	3	2n	*O* II 5.87(10)?; *Si* II 5.45(2)
3970.17	3	7	*He* 0.08(6)				
3972.81	1	1	*O* II 3.27(10)?	4077.18	2#	1n	*Sr* II 7.71(400r); *Si* II 6.78(1)
3974.09	1	1	*Fe* II 4.17 pred.				
3976.96	2	1	.....	4079.69	2	1	.....
3979.63	2	1	*S* II 9.81(4); *C* II 80.35(2); *Cr* II 9.51(2)	4082.36	2	1	*N* II 2.28(2n)
				4101.95	3	8	*Hδ* 1.74(7)
				4109.65	2	1	.....
3983.58	1	1	*S* III 3.76(7)?	4111.15	2#	1	*Cr* II 1.00 (1)
3985.49	1	1	*S* III 5.97(5)?	4113.34	1##	1	*Cr* II 3.23(1)
3991.27	2	1n	*S* II 0.90(5); *Si* II 1.77(2n)	4115.05	3	1	.....
				4117.29	3	1+	.....
3992.93	2	1	*S* II 3.49(6)	4120.76	3	4	*He* I 0.81(3)
3994.95	3	3	*N* II 5.00(10)	4122.76	3	2n	*Fe* II 2.67 pred.
3998.13	2	2	*S* II 8.74(5); *Si* II 8.00(1n)	4127.96	2	7	*Si* II 8.05(8)
				4129.37	2	1	.....
4000.34	2	1	.....	4130.86	3	8	*Si* II 0.88(10); *S* II 0.90(2)
4002.51	3	2n	†				
4003.78	1	1	*S* II 3.87(2); *Cr* II 3.33(2)	4132.96	2#	2	*N* II 3.65(2); *Cr* II 2.40(1)
4005.34	2	2n	*V* II 5.71(60)?; *Fe* I 5.25(25)	4135.49	1##	2	.....
				4143.48	3	5	*He* I 3.76(2); *S* II 2.24(8); *S* II 5.05(8)
4009.27	3	7	*He* I 9.27(1)				
4011.61	2	2	.....	4146.93	1#	1	*S* II 6.90(5)
4013.88	2	2n	.....	4150.12	2#	1	.....
4015.56	3	2n	*Ni* II 5.50(1)	4152.99	2	2	*S* II 3.05(10); *Cr* II 3.07(4)
4018.08	2	1	.....				
4026.39	3	8	*He* I 6.19(5)	4157.15	3	1	.....
4028.75	2	1n	*S* II 8.74(7)	4159.89	2#	1	.....
4030.64	2	1	.....				

* The H line of *Ca* II was measured on only one plate; on the other two plates it is blended with the B5 component of *Hε*.

† This line was identified with *Fe* II by Baxandall, *Solar Physics Committee*, **2**, 33, 1910.

TABLE I—*Continued*

Star I.A.	No. of Plates	Intensity	Identification	Star I.A.	No. of Plates	Intensity	Identification
4162.46..	3	3	*S* II 2.64(10); *S* II 2.29(2)	4269.58..	3	1+	*S* II 9.72(5); *Cr* II 9.28(1)
4164.98..	3	1n	*Ti* II 3.65(40); *S* II 5.20(3); *S* II 4.98(2)	4271.25..	2	1	*Fe* I 1.76(35)
				4273.36..	3	2	*Fe* II 3.31(1)
				4275.69..	2	1	*Cr* II 5.55(1)
4166.50..	2	1	.....	4278.65..	2	1n+	*S* II 8.51(4)
4168.78..	3	3	*He* I 8.97(1); *S* II 8.37(5)	4279.82..	2	1n+	.....
				4282.11..	3	2	*S* II 2.60(4)
4171.48..	3	1	*N* II 1.63(2n); *Ti* II 1.91 (30)	4284.66..	2	1+	*S* III 4.99(8)?; *Cr* II 4.20(2)
				4288.76..	2	1	.....
4173.69..	3	3	*Fe* II 3.47(5); *S* II 3.97(4); *S* II 4.25(5)	4290.48..	2#	1	*Ti* II 0.23(50)
				4291.78..	2	1	*S* II 1.43(1)
				4294.26..	3	3n	*Ti* II 4.10(40); *S* II 4.39(6)
4176.04..	3	1	*N* II 6.17(3n)				
4178.84..	3	3	*Fe* II 8.87(8); *Cr* II 9.34(2)	4296.73..	3	3n	*Fe* II 6.56(3)
				4300.00..	3	2nn+	*Ti* II 0.05(60)
4182.08..	3	1	.....	4303.14..	3	3	*Fe* II 3.18(4)
4187.01..	2#	1n	.....	4305.62..	2	1	*Sc* II 5.71(6)
4189.86..	2#	1n	*O* II 9.79(10)?; *S* II 9.68(4); *Si* II 90.74(3)	4307.81..	3	2n	*Ti* II 7.89(40); *Fe* I 7.91(35)
				4309.99..	3	1	.....
4193.19..	2#	1	.....	4314.62..	2#	2+	*Ti* II 4.98(40); *Sc* II 4.09 (30)
4196.10..	1##	1	.....				
4198.02..	1##	1n	*Si* II 8.17(2)				
4200.49..	3	2n	‡	4318.09..	2	1	*S* II 8.64(4)
4205.52..	3	1	*V* II 5.09(30)?	4320.89..	2	1+	*Sc* II 0.73(20)
4210.55..	2	1	.....	4325.68..	3	2n	*Fe* I 5.77(35); *Sc* II 5.00 (20)
4215.71..	1#	1	*Sr* II 5.52(300r)				
4224.92..	2	1	*Cr* II 4.85(2)				
4227.56..	3	1+	*N* II 7.83(3n)	4328.74..	2	1+	.....
4230.49..	3	2n	*S* II 0.94(4)	4340.95..	3	9	*H*γ 0.46(8)
4233.11..	3	6	*Fe* II 3.16(10)	4351.77..	3	4	*Fe* II 1.77(10)
4236.71..	3	2n+	*N* II 6.98(6n)	4354.64..	3	1+	*S* III 4.57(5)?; *Sc* II 4.60(5)
4242.27..	3	3n	*Cr* II 2.39(5); *N* II 1.80(8n)				
				4357.79..	3	2	.....
4244.68..	2	1n	*Ni* II 4.80(1)	4362.08..	3	1+	*Ni* II 2.10(1); *S* III 1.54(7)?
4246.91..	2#	1n+	*Sc* II 6.84(100)				
4249.98..	3	1n+	*S* II 9.94(1)	4367.27..	2	1	*Ti* II 7.67(15)
4253.18..	3	1n+	*S* III 3.59(10)?; *Cr* II 2.62(1)	4369.41..	3	1	*Fe* II 9.41 pred.
				4372.04..	1	1	*C* II 2.49(1)
4255.39..	1#	1	.....	4374.13..	1	1	*C* II 4.28(2); *Sc* II 4.46(30)
4256.62..	1#	1	.....				
4258.23..	3	2	*Fe* II 8.16 pred.	4375.25..	2	1n	.....
4261.90..	3	1	*Cr* II 1.92(2)	4376.95..	1	1	.....
4265.06..	3	1n	.....	4379.54..	2	1	.....
4267.30..	3	3n	*C* II 7.27(10); *C* II 7.02(8); *S* II 7.27(4); *S* II 6.90(4); *S* II 7.76(6)	4384.80..	3	5	*Fe* II 5.39(5); *Mg* II 4.64(8)
				4388.11..	3	6	*He* I 7.93(3)
				4390.55..	3	3	*Mg* II 0.58(10)
				4392.73..	2	1	.....

‡ One of the strongest unidentified lines in stellar spectra.

TABLE I—*Continued*

Star I.A.	No. of Plates	Intensity	Identification	Star I.A.	No. of Plates	Intensity	Identification
4395.42..	3	3n	*Ti* II 5.04(60)	4519.99..	3	3	*Fe* II 20.24(8)
4397.96..	2	1	.....	4522.58..	3	3+	*Fe* II 2.64(10)
4400.22..	3	2n	*Ti* II 9.77(35); *Sc* II 0.38(20)	4524.85..	3	2n	*S* II 4.96(6); *S* II 4.65(2)
4402.63..	1	1	*S* II 2.64(2)	4529.24..	2	2n	*Al* III 9.18(6)
4404.15..	3	2	.....	4531.52..	3	1	.....
4409.00..	3	1	.....	4534.19..	3	3n	*Fe* II 4.17 pred.; *Mg* II 4.26 (4); *Ti* II 3.97(30)
4411.26..	2#	1+	*C* II 1.20(2); *C* II 1.52(2)				
4414.19..	2	1	*O* II 4.89(10)?				
4416.85..	3	4	*Fe* II 6.81(8)	4536.84..	2	1	.....
4419.76..	3	2+	‡	4539.58..	2	1	.....
4421.40..	1	1	.....	4541.91..	3	2	*Fe* II 1.52 pred.
4422.83..	2	1	.....	4543.01..	2	1	.....
4424.71..	1	1	.....	4546.01..	3	1	.....
4427.80..	3	2n	*Mg* II 7.99(7); *N* II 7.97(2); *N* II 7.21(2)	4549.35..	3	5	*Fe* II 9.48(20); *Ti* II 9.64 (60); *S* II 9.56(4)
4430.83..	3	2	*S* II 1.00(2)				
4434.17..	3	2nn	*Mg* II 3.99(8); *N* II 3.48(2n); *N* II 2.71(6n)	4552.51..	3	3+	*Si* III 2.61(9); *N* II 2.50(4); *S* II 2.37(5)
4437.61..	3	3+	*He* I 7.55(1)	4555.74..	3	4	*Fe* II 5.90(15); *Cr* II 4.99(3)
4440.67..	3	1+	.....				
4443.86..	3	1n+	*Ti* II 3.80(50)	4558.72..	3	3+	*Cr* II 8.64(20)
4446.78..	2	2	*N* II 7.04(10)	4563.55..	2#	2	*Ti* II 3.77(30)
4450.13..	3	1n	*Ti* II 0.49(10)	4567.80..	2	2	*Si* III 7.83(7)
4451.92..	3	1n+	.....	4571.91..	3	2n	*Ti* II 1.98(50)
4455.91..	3	2	*S* II 6.39(4)	4576.28..	3	3	*Fe* II 6.31(10)
4460.39..	3	1	.....	4574.47..	2	1	*Si* III 4.75(4)
4461.70..	1	1n	.....	4579.98..	3	2	.....
4463.47..	2	1n	*S* II 3.58(7)	4583.35..	3	5	*Fe* II 3.84(30); *Fe* II 2.84 pred.
4471.67..	3	6	*He* I 1.48(6)				
4474.14..	2	1+	.....				
4478.23..	2	1+	.....	4588.22..	3	2	*Cr* II 8.20(15); *Al* II 8.19(5)§
4481.14..	3	10	*Mg* II 1.13; *Mg* II 1.33 (100)	4591.96..	3	2	*Cr* II 2.05(2)
				4595.62..	3	2	*Fe* II 5.70 pred.
4483.90..	3	1+	*S* II 3.42(6)	4601.55..	3	1+	*N* II 1.49(8)
4486.54..	2	1n+	*S* II 6.63(3)	4604.63..	2	1	.....
4488.93..	3	2n	*Fe* II 9.21(10)	4607.20..	2	2	*N* II 7.17(7)
4491.39..	3	2n	*Fe* II 1.41(8)	4609.87..	3	2	.....
4496.98..	2	1	.....	4613.83..	3	2	*N* II 3.88(6)
4499.07..	1	1	*Cr* II 8.73(6)	4616.95..	3	2	*Cr* II 6.62(3r)
4500.16..	2	1	.....	4619.18..	1##	2	*Cr* II 8.81(10)
4501.36..	1	1	*Ti* II 1.27(40)	4621.44..	3	2	*N* II 1.41(7)
4503.46..	3	1	.....	4626.09..	2	1+	.....
4508.17..	3	4n	*Fe* II 8.29(15)	4630.08..	3	3	*Fe* II 29.33(20); *N* II 30.55 (10)
4512.37..	3	2	*Al* III 2.54(4)				
4515.27..	3	3+	*Fe* II 5.34(15)				
4517.82..	2	1+	.....	4634.61..	3	2+	*Cr* II 4.07(10)

‡ One of the strongest unidentified lines in stellar spectra.

§ Identified by Struve, *Astrophysical Journal*, **74**, 237, 1931, with *Al* II in β Orionis.

TABLE I—*Continued*

Star I.A.	No. of Plates	Intensity	Identification	Star I.A.	No. of Plates	Intensity	Identification
4638.07..	2#	1+	*Si* III 8.12(1)	4667.09..	2	1	*N* II 7.28(2)
4643.09..	3	2	*N* II 3.11(8)	4678.06..	1	1	*N* II 7.93(3n)
4647.28..	1	1	*S* II 7.48(2)	4682.39..	2	1	*Si* II 3.02(2)
4650.22..	2	1+	.....	4707.96..	1	1	.....
4656.97..	3	2	*S* II 6.75(5)	4713.87..	1##	5	*He* I 3.14(3);
4663.23..	3	2	*Al* II 3.05(10)§				*He* I 3.37(1)

§ Identified by Struve, *Astrophysical Journal*, **74**, 237, 1931, with *Al* II in β Orionis.

ELEMENTS PRESENT IN THE B8 SPECTRUM OF β LYRAE

H	$H\gamma$, $H\delta$, $H\epsilon$
He I	12 lines, 10 unblended
Ca II	2 lines (H, K)
Si II	9 lines, 3 unblended
Si III	4 lines, 3 unblended
Mg II	5 lines, 2 unblended
Fe I	6 lines, 3 unblended
Fe II	27 lines, 19 unblended
Ti II	17 lines, 10 unblended
N II	25 lines, 18 unblended
Ni II	4 lines, 3 unblended
Cr II	26 lines, 14 unblended
C II	6 lines, 2 unblended
S II	40 lines, 23 unblended
S III	6 lines, identification doubtful
Sr II	2 lines, 1 line unblended
O II	4 lines, presence uncertain
V II	4 lines, presence uncertain
Al II	2 lines, 1 unblended
Al III	2 lines, 2 unblended
Sc II	7 lines, 3 unblended

lengths and identifications of the B8 absorption spectrum. The observed wave-lengths in column 1 are the averages of separate determinations. Column 2 gives the number of plates (3, 2, or 1) upon which the line was measured. The symbols # and ## indicate that the line in question was later suspected on either one or two enlargements of the plates. Column 3 gives the estimated intensity of the line. Intensity 1 indicates that the line is barely visible, while intensity 10 was assigned to the *Mg* II line λ 4481. Column 4 gives the identifications, with the laboratory wave-lengths and intensities. The following sources were used in making these identifications:

Charlotte E. Moore. "A Multiplet Table of Astrophysical Interest" (1933).
O. Struve. "A Study of the Spectra of B Stars."[2]
D. H. Menzel. "A Study of the Solar Chromosphere."[3]

A comparison of the number of lines measured with the number identified by R. H. Curtiss,[4] by F. E. Baxandall,[5] and by myself, is

[2] *Astrophysical Journal*, **74**, 229, 1931.
[3] *Lick Observatory Publications*, **17**, Part V, 1931.
[4] *Publications of the Allegheny Observatory*, **2**, 73, 1912.
[5] *Annals of the Solar Physics Observatory, Cambridge*, **2**, Part I, 1930.

shown in the accompanying table. The increase in the number of lines which I have identified is due to weaker lines of elements, such

	No. Meas.	No. Ident.
Curtiss	75	52
Baxandall	69	67
Pillans	240	167

as *Cr* II, *Si* II, *Sc* II, *Mg* II, and *Fe* II, and to the new identification of 40 lines with *S* II, 4 lines with *Si* III, and 25 lines with *N* II. Among the *N* II lines only one, λ 3995, had been identified by Baxandall.

On comparing the spectrum of the B8 component of β Lyrae with those of the supergiants β Orionis (B8p) and σ Cygni (A0p), it appears that the *H* lines are slightly weaker in β Lyrae than in β Orionis, and a great deal weaker than in σ Cygni. The *He* I lines are of about the same intensity as in β Orionis, and are much stronger than in the other comparison star. In his work on the B-type stars, O. Struve identified sixteen lines in the spectrum of β Orionis with *O* II. In β Lyrae the four strongest lines, of intensity 10 in the laboratory, are doubtfully present. The *Mg* II line, λ 4481, and the *Si* II doublet, λ 4128 and λ 4130, have practically the same intensity in all three stars. The *Si* III triplet, λ 4552, λ 4567, and λ 4547, is stronger in β Lyrae than in β Orionis, while in σ Cygni only the strongest member of the triplet, λ 4552, appears. The lines of *Fe* II and the stronger lines of *Ti* II are appreciably stronger in β Lyrae than in β Orionis, but are weaker than in σ Cygni. Finally, three of the strongest *Fe* I lines are blended in β Lyrae while only two of them have been doubtfully measured in β Orionis.[6] From these comparisons it seems that the B8 component of β Lyrae is normal for its class, with, however, unusually strong metallic lines, which perhaps signifies that it may belong to a slightly later spectral class. Although the complexity of the *H* lines and of the region around λ 4470 makes it impossible to apply two of the criteria used in separating giants from dwarfs,[7] the strength of the metallic lines and the appearance of *He* I lines belonging to the diffuse series are sufficient to place it in the supergiant class.

[6] W. W. Morgan, unpublished material.

[7] Struve, *op. cit.*, p. 249.

In Table II a comparison between the number and intensity of lines in 67 Ophiuchi (B5) and in the B5 component of β Lyrae is given. Only the lines in 67 Ophiuchi whose intensities were estimated as 2 or greater have been included in the list, but it is immediately apparent that there are a great many normal B5 lines missing from the B5 spectrum of β Lyrae. With the exception of *Si* II 4128,[8] *Mg* II 4481, and perhaps *C* II 4267, all of the lines belong

TABLE II

Lab. λ	Identification	67 Ophiuchi Intensity	B5 Component of β Lyrae		Lab. λ	Identification	67 Ophiuchi Intensity	B5 Component of β Lyrae	
			λ	Intensity				λ	Intensity
3933.66	*Ca* II	6	.....	....	4162.73	*S* II	4	.....	....
3935.91	*He* I	2	.....	....	4168.97	*He* I	4	.....	....
3964.73	*He* I	8	3964.16	10	4233.16	*Fe* II	1	.....	....
3968.46	*Ca* II	8	.....	....	4267.27	*C* II	8	Possibly present	....
3970.08	*Hε*	15	3970.21	8					
3995.00	*N* II	4	.....	....	4340.47	*Hγ*	20	4340.48	10
4009.27	*He* I	9	4009.54	2	4387.93	*He* I	9	4388.15	10
4015.50	*Ni* II	1	.....	....	4437.55	*He* I	2	.....	....
4023.99	*He* I	2	.....	....	4447.04	*N* II	2	.....	....
4026.19	*He* I	12	4026.40	8	4471.48	*He* I	10	4471.38	10
4041.32	*N* II	2	.....	....	4481.33	*Mg* II	8	4481.57	2
4101.74	*Hδ*	20	4101.82	10	4549.48	*Fe* II	2	.....	....
4120.81	*He* I	7	4120.61	2	4552.65	*Si* III	5	.....	....
4128.05	*Si* II	8	4127.93	2	4567.87	*Si* III	4	.....	....
4130.88	*Si* II	8	.....	....	4574.78	*Si* III	3	.....	....
4142.31	*S* II	2	.....	....	4713.14	*He* I	8	Not measured	8
4143.77	*He* I	9	4143.72	2					
4145.16	*S* II	2	.....	....					
4153.31	*O* II	2	.....	....					

to *H* and *He* I; so that, therefore, the B5 component of β Lyrae is definitely not normal for its class.

II

On a plate taken $0^d.38$ after principal minimum, the usual emission lines of *H* and *He* are exceptionally strong and, in addition, there appear a number of other emission lines which cannot be seen on the other plates. This plate of β Lyrae was compared with a Process

[8] If *Si* II 4130 also belongs to the B5 spectrum, it would fall upon *Si* II 4128 during the phases when the two components are well separated.

plate of P Cygni (B2) and the new lines were found to coincide with previously identified emission lines in this star. The results of this comparison are given in Table III. The wave-lengths, identifications, and intensities for lines in the spectrum of P Cygni are those given by Struve.[9] All of the emission lines in β Lyrae are about twice as wide as those in P Cygni. An examination of the table

TABLE III

Identification	λ	Intensity in P Cygni	Intensity in β Lyrae	Identification	λ	Intensity in P Cygni	Intensity in β Lyrae
H	4101.95	10	5	*N* II	4447.07	2	.....
	4340.95	10	5		4601.49	4	Trace
He I	4026.39	5	5		4607.17	4	Trace
	4120.81	4	3		4613.88	3	.....
	4143.77	5	3		4621.30	3	4
	4387.93	5	4		4630.55	5	5
	4437.55	1	.....		4643.15	5	4
	4471.48	10	10				
	4713.87	5	5	*N* III	4097.31	2	.....
O II	4366.91	1	.....	*C* II	4267.27	1	.....
	4414.89	2	.....				
				Mg II	4481.23	1	5
Si III	4552.65	2	.....				
				Unidentified	4382.43	3	.....
Si IV	4116.10	1	.....		4395.95	3	.....
					4419.62	4	2
					4431.03	3	.....

shows that the emission spectrum of β Lyrae is probably of a later type than that of P Cygni; lines due to *Si* III, *Si* IV, and *N* III, which are present in P Cygni, are absent from β Lyrae. The other lines are, for the most part, comparable in intensity with the lines in P Cygni; lines of intensity 4 in P Cygni are apparently at the limit of visibility in β Lyrae. *Mg* II 4481 and *N* II 4621 are, however, two very conspicuous exceptions, since both of these lines, although they are relatively weak in P Cygni, are very strong in β Lyrae.

YERKES OBSERVATORY
March 8, 1934

[9] *Op. cit.*, **76**, 100, 1932.

NOTES

THE PRESENCE OF PHOSPHORUS IN THE SUN

ABSTRACT

On the basis of spectroscopic evidence phosphorus may with some assurance be added to the long list of elements present in the sun. Three infra-red solar lines are attributed without question to phosphorus, and two with some doubt.

The laboratory spectrum is discussed and a table gives the observable multiplets and corresponding solar data from which the results are derived.

Since the spectrum of phosphorus consists of two widely separated groups of lines—one astronomically inaccessible, of wave-length shorter than λ 2700, and the other beyond λ 9100—a search in the solar spectrum for lines due to this element is necessarily restricted to the infra-red region.

Table I contains the laboratory and solar data for the multiplets

TABLE I

Laboratory						Sun		
λ I.A.	Lab. Int.	Theor. Int.	Lower E.P.	I.Q. No.	Multiplet Desig.	Disk Int.	Δλ ⊙−Lab.	Id.
9175.85±.03	0	10	6.91	$\frac{1}{2}$–$1\frac{1}{2}$	$4s^4P-4p^4S^\circ$	−3	+.18	.....
9304.88±.03	3	20	6.92	$1\frac{1}{2}$–$1\frac{1}{2}$	.....	.....	Abs.	.....
9525.75±.03	30	30	6.96	$2\frac{1}{2}$–$1\frac{1}{2}$	.....	−3	+.20	.....
9563.45±.03	12	27	6.92	$1\frac{1}{2}$–$2\frac{1}{2}$	$4s^4P-4p^4P^\circ$	.....	Abs.	.....
9593.54±.03	25	25	6.91	$\frac{1}{2}$–$1\frac{1}{2}$	.....	.....	Abs.	.....
9608.97±.03	3	5	6.91	$\frac{1}{2}$– $\frac{1}{2}$	.....	−2	−.05	.....
9734.74±.03	20	8	6.92	$1\frac{1}{2}$–$1\frac{1}{2}$	.....	.....	Abs.	.....
9750.73±.03	25	25	6.92	$1\frac{1}{2}$– $\frac{1}{2}$	.....	.....	Abs.	.....
9796.76±.03	50	63	6.96	$2\frac{1}{2}$–$2\frac{1}{2}$	.....	−3	−.03	P
9976.65±.03	5	27	6.96	$2\frac{1}{2}$–$1\frac{1}{2}$	.....	.....	Abs.	.....
10511.43±.04	3	25	6.91	$\frac{1}{2}$–$1\frac{1}{2}$	$4s^4P-4p^4D^\circ$	.....	Abs.	.....
10529.45±.04	6	63	6.92	$1\frac{1}{2}$–$2\frac{1}{2}$	.....	−1	−.01	P
10581.52±.04	8	120	6.96	$2\frac{1}{2}$–$3\frac{1}{2}$	.....	−1	+.08	P
10596.92±.04	1	25	6.91	$\frac{1}{2}$– $\frac{1}{2}$	.....	−3	−.03	P?
10681.43±.04	1	32	6.92	$1\frac{1}{2}$–$1\frac{1}{2}$	.....	−3	+.01	P?
10813.03±.04	0	27	6.96	$2\frac{1}{2}$–$2\frac{1}{2}$	.....	−3	+.15	.....
9790.08±.03	3	.....	7.15	$\frac{1}{2}$–$1\frac{1}{2}$	$4s^2P-4p^2P^\circ$	−3	−.13	.....
9903.74±.03	8	.....	7.15	$\frac{1}{2}$– $\frac{1}{2}$	.....	−3	−.07	.....
10084.22±.04	25	.....	7.18	$1\frac{1}{2}$–$1\frac{1}{2}$	.....	oN	+.19	m*
10204.72±.04	2	.....	7.18	$1\frac{1}{2}$– $\frac{1}{2}$	.....	.....	Abs.	.....

*m = masked.

most likely to be represented if phosphorus can be detected spectroscopically in the sun. Kiess,[1] who has analyzed the spectrum, states that his estimated intensities "recorded for lines in different portions of the spectrum are not comparable with each other." For convenience the theoretical intensities reduced to a uniform scale have, therefore, been entered in column 3.

The sun *minus* laboratory residuals in wave-length are on the whole no more inconsistent than those of lines due to other elements which are known to be present. As yet, considerable tolerance must be allowed for infra-red laboratory measures. Since the three lines which should theoretically be the strongest are so close to unidentified solar lines, the identifications here suggested seem real. The agreement of the faint line at λ 9608 with a solar line may be regarded as accidental. The absence of λ 9525, as apparently indicated by the large residual, is not serious; λ 9525 is considerably fainter than the three lines attributed without question to phosphorus; and λ 10084 is probably masked by a diffuse solar line. Although highly accurate infra-red laboratory wave-lengths are desirable, yet on the present evidence phosphorus can with some assurance be classed among the elements present in the sun.

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[1] *Bureau of Standards Journal of Research* **8**, 393 (RP 425), 1932.

PHOTO-ELECTRIC OBSERVATIONS OF THE COLOR OF THE ZODIACAL LIGHT

ABSTRACT

The color of the zodiacal light has been measured on five nights with a photo-electric photometer and has been found to be variable. The mean color is nearly that of the sun, which indicates that the scattering of the sunlight is by large particles.

According to the generally accepted theory, the zodiacal light is produced by the reflection of sunlight from myriads of particles concentrated near the plane of the ecliptic and extending out to and beyond the orbit of the earth. Its spectrum should be similar to that of the sun; this has been found to be the fact by E. A. Fath,[1] who used a spectrograph of low dispersion. If the particles are large, the scattering of the sunlight would be independent of the wave-length. On the other hand, if the particles are of sizes of the order of one-quarter of the wave-length of the incident light, or less, the scattering would follow Rayleigh's law, that is, would be inversely proportional to the fourth power of the wave-length. Particles of intermediate size would produce scattering which would be proportional to the inverse first, second, or third powers of the wave-length; but, as E. Schönberg[2] has shown, the range in size of these particles is small, so that for a uniform mixture of particles of all sizes these intermediate powers may be neglected. With the photo-electric photometer of the Yerkes Observatory as used for colorometric work, the theoretical difference in color between the incident light and that scattered according to the Rayleigh law is 0.5 mag.[3] The effective wave-lengths of the pair of filters (Y_1 B_1) when used with a sensitized potassium cell in this photometer are 4750 and 4250 A, respectively. Therefore, it should be easy to demonstrate whether the sunlight scattered by the particles obeys the law of reflection or of Rayleigh scattering.

Observations of the color of the zodiacal light in the morning sky were begun in October, 1933, but owing to poor observing weather only two mornings could be used. In this work only the best of nights for photometric observations were employed, since it is necessary to work at large zenith distances to make measures in the in-

[1] *L.O.B.*, **5**, 141, 1909.

[2] *Mitt. d. Sternwarte Breslau*, **3**, 53, 1932.

[3] O. Struve, C. T. Elvey, and P. C. Keenan, *Ap. J.*, **77**, 274, 1933.

tense part of the zodiacal light. Three more observations of the evening zodiacal light were obtained in February and March. In each case except one, February 14, the color of the zodiacal light was determined with respect to a comparison star. The procedure was to observe the color of a standard star, η Tauri, at the beginning of the observations, then the comparison star, then the zodiacal light as near to it as possible, then the comparison star again, and lastly the standard star and other stars for final comparison. If time allowed, observations of the light were made on each side of the comparison star. This was done for the morning zodiacal light in October when it was very bright and visible for a long time before dawn. On October 17 the extinction factor was determined from a series of observations of β Tauri, and on February 14 from observations of η Tauri. The colors of the extinction stars were plotted against sec z, and the factor E in the formula $C_0 = C - E$ sec z was determined from a straight line drawn through the observations. The extinction for the other nights was determined from the standard star, η Tauri, under the assumption that its color is that determined on February 14. Several additional stars were observed for comparison.

The results of the observations are shown in Table I. The color of the zodiacal light and that of the comparison star are printed in italics. February 14 was the only night on which a direct comparison star was not used. On the morning of October 17 two direct comparisons were made, first with α Leonis and then with σ Leonis. The agreement is very satisfactory. The next observation was made on the night of poorest general transparency of the sky during the series, and only one comparison was made, namely, with σ Leonis. It is seen that the color of the zodiacal light has increased by 0.4 mag. and that of the comparison star by 0.15 mag. The next observation was not made until the evening of February 14, and was without a direct comparison. Again the color is larger. On the evenings of March 9 and 10 the color of the zodiacal light was compared directly with the star o Piscium, of spectral type K0. The first night gave a color agreeing with that obtained on October 17, but the next night gave the largest color of all the observations.

These variations in color of the zodiacal light seem too large to be ascribable to errors of observation, even at such large zenith dis-

tances. A study of the original observations and of the comparison stars shows that a large part of the variation is real. A significant point is the direct comparison with the star *o* Piscium. On the night of March 9 the zodiacal light was 0.4 mag. bluer than this K-type star, and on the next night it was 0.1 mag. redder. A part of the variation, however, could be due to local changes in the color of the sky radiation. The light of the sky alone may contribute up to 50 per cent of the total light observed. All of the observations were made

TABLE I

		Evening, 1933			Morning, 1934		
Object	Sp.	Oct. 17	Oct. 17	Oct. 24	Feb. 14	March 9	March 10
Zodiacal light	.....	+0.38	+0.34	+0.75	+0.93	+0.28	+1.03
α Leonis	B8	.042	.....	.086	.....	.047	0.038
σ Leonis	A0	.....	+0.05	.19	.....	.....	.....
η Tauri	B5p	.017	.....	*	.070	*	*
α Pegasi	A0	.054	.....	.....	.....	.....	.....
α Aurigae	G0	.622	.....	.582	+0.585	.610	.598
o Piscium	K0	.....	.....	.....	.....	+0.70	+0.92
β Tauri	B8	+0.018	.....	+0.023	.....	.....	.....
Extinction factor	.....	0.100	0.100	0.060	0.098	0.084	0.080

*The color of η Tauri was assumed as +0.070 for these nights, in order to determine the extinction factor.

sufficiently far from the time of twilight that no difficulty from it can be expected.

Neglecting the large variations in the color of the zodiacal light and taking the mean, we find the color to be +0.62 mag., with a mean error of ±0.29 mag. This color is very nearly that of α Aurigae, a star of about the same spectral class and temperature as the sun. Since the color of the zodiacal light would have to be 0.5 mag. bluer than that of the sun if it were due to Rayleigh scattering, we may conclude that the light is produced by reflections from large particles.

It would be very desirable to obtain simultaneous observations of the color and the spectrum of the zodiacal light, if a sufficiently rapid spectrograph could be had.

C. T. Elvey

Yerkes Observatory
May 10, 1934

THE FORBIDDEN HELIUM LINE λ 4470

ABSTRACT

Several arguments are given in favor of the identification of the stellar absorption line at λ 4470 with the forbidden helium line 2p−4f.

In a paper presented last summer at the Chicago meeting of the American Physical Society, J. S. Foster and A. V. Douglas[1] reported that the observed stellar line λ 4470 in certain B-type stars—for example, γ Pegasi—may not be due entirely to the forbidden transition 2p−4f. Their calculations showed that the true forbidden line should be relatively weak, and they identified it with a well-defined portion of the heavy complex observed line which otherwise remained unknown. Similarly, R. K. Marshall[2] expressed the opinion that the lines identified by me with forbidden helium may not be due to that atom alone.

The objections may be summarized as follows: (1) The wave-length of the observed stellar line at λ 4470 agrees almost exactly with that of the forbidden helium line computed for a field intensity zero. In reality an appreciable shift toward the violet should have been expected for the field intensities considered probable. (2) The total intensity of the stellar line is in excess of that shown by laboratory experiments in emission for probable field intensities.

While both objections are valid, there are a number of reasons why the identification of the stellar line with forbidden helium remains probable. These are:

1. The existence of Stark effect shown by the broadening of members of the diffuse helium series is beyond question and has apparently been confirmed by several other observers. It can be very clearly seen in my reproductions of stellar spectra, published in this *Journal*, **74**, 225, 1931. It is even more clearly brought out in Marshall's spectrograms obtained at Ann Arbor, copies of which he has kindly sent me.

2. The permitted helium lines which are subject to Stark broadening show little or no displacement in wave-length. This may be due to the absorption in higher levels where the pressures are low.

[1] *Phys. Rev.*, **44**, 325, 1933.

[2] *Publs. of the Observatory of the University of Michigan*, **5**, 137, 1934.

It is not especially surprising that no shift is shown by the forbidden line.

3. The contour of the stellar line λ 4470 is broad and diffuse, resembling the permitted lines of helium which are subject to Stark broadening. This has been shown quantitatively in a paper by J. Pauwen.[3] The lines of other elements, except hydrogen, are normally sharp and narrow. It is, therefore, not possible to identify the stellar line with another element.

4. Nearly all strong lines in B-type stars have been satisfactorily identified. It would be an almost unbelievable coincidence if a strong unidentified line should exist in the exact position of the forbidden helium line.

5. The stellar line λ 4470 shows a distinct effect of absolute magnitude. It is strongest in the dwarfs and weakest in the giants. This is contrary to the effect shown by most other elements. Thus lines of oxygen are generally stronger in the giants than in the dwarfs.

6. The stellar line reaches maximum intensity at exactly the same place in the spectral sequence as do the permitted lines of helium. The atom which gives rise to λ 4470 must, therefore, have characteristics which are similar to the atom of helium. The atoms in the periodic table which satisfy this requirement do not possess lines at that wave-length.

7. Relative intensities measured in the laboratory are not directly applicable to stellar absorption lines. It is well known that many predicted lines of chromium, iron, and other elements, though never observed in the laboratory, are fairly strong in the stars. There is at present no definite rule for converting laboratory emission intensities into stellar absorption intensities. The existence of the gradient effect, discussed by Elvey and Struve,[4] makes it practically impossible to derive any conclusions as to the expected stellar intensities.

OTTO STRUVE

YERKES OBSERVATORY
June 14, 1934

[3] *Ap. J.*, **70**, 263, 1929.

[4] *Ibid.*, **79**, 409, 1934.

REVIEWS

Handbuch der Astrophysik. Edited by G. EBERHARD, A. KOHLSCHÜTTER, and H. LUDENDORFF. 6 vols. Vol. I: *Foundations of Astrophysics.* Pp. xii+464; Figs. 299. Berlin: Julius Springer, 1933. RM. 96; bound, RM. 99.

CONTENTS

"Foundations of Theoretical Optics," by H. SCHULZ (Berlin): Polarization, reflection, and refraction. Scattering of light by small particles. Interference phenomena. Michelson's interferometer. Lummer-Gehrcke interference spectroscope. Fabry-Perot plates. Diffraction. Doppler effect and its experimental proofs. Zeeman effect and Stark effect with application to solar and stellar spectra.

"The Telescope," by ALBERT KÖNIG (Jena): Geometrical optics. The image as a diffraction phenomenon. The construction of telescopes. Astronomical objectives. Objectives for spectrographs. Properties of different glasses. Eyepieces. Concave mirrors. Reflecting telescopes. The testing of telescopes. Twyman's interferometer. Methods by Waetzmann, Ronchi, Hartmann, etc.

"Spectroscopy," by C. RUNGE (Göttingen) and K. W. MEISSNER (Frankfurt a/M): Theory of refraction of light by prisms. Minimum deviation. Various types of prisms. Astigmatism and its avoidance. Curvature of spectral lines. Theory of the grating and of interference spectroscopes. Systems of wave-lengths. Rowland's system. Michelson's interferometer measurements. Pressure shifts. The international angstrom unit.

"Stellar Spectroscopy and the Determination of Radial Velocities," by G. EBERHARD (Potsdam): The construction of stellar spectrographs. Mechanical stability. Temperature control. The optical parts. Description of the telescope and spectrograph of the Dominion Astrophysical Observatory at Victoria, B.C. Mechanical construction of spectrographs by W. H. Wright, by J. S. Plaskett, and by P. W. Merrill. Spectrograph objectives. Systematic errors in the determination of radial velocities. Methods of reduction by Cornu, Hartmann, etc. Stellar wavelength table. Correction of radial velocities to the sun. Determination of radial velocities with the objective prism.

"The Measurement of the Total Radiation of Celestial Objects," by W. E. BERNHEIMER (Vienna): Introduction. Selective measurements in the infra-red. The actinometer and the pyrheliometer for measuring the total radiation of the sun. The bolometer and the spectrobolometer. The pyranometer and its astrophysical applications. The determination of the solar constant. The bolometer and the thermopile for investigating sunspots and the corona. The application of thermopiles for the measurement of the ultra-violet solar radiation. The measurement of the total radiation of planets and stars. The use of the bolometer for measuring the energy distribution in stellar spectra. The determination of the energy spectra in stars with the radiometer. Possible improvements in the instruments.

"The Determination of Star Positions from Direct Photographs," by ARTHUR KÖNIG (Jena): Introduction. The photographic procedure and the measurement of the plates. Tangential coordinates and their transformation. Instrumental corrections. Scale-value, orientation, zero-point, inclination. Refraction. Aberration. Precession and nutation. Reference stars. Tables.

Ibid., Vol. II, Part I: *Foundations of Astrophysics*. Pp. xi+430; Figs. 134. 1929. RM. 59.40; bound, RM. 62.10.

CONTENTS

"Theoretical Photometry," by E. SCHOENBERG (Breslau): Introduction. Definitions and fundamental laws. Diffuse reflection. The illumination of planets. The illumination of satellites. The effect of diffraction in the telescope upon the distribution of light on the disk of a planet. The apparent diameters of planets. The illumination of dust clouds. The extinction of light in the atmosphere of the earth. Scattering and absorption of light in gases. Application of the theory to the atmospheres of planets. Photometric tables.

"Spectrophotometry," by A. BRILL (Neubabelsberg): The optical methods. The spectrophotometer and its applications to the determination of the energy distribution in a spectrum. The determination of the ratios in intensity in different sources. Visual methods for the determination of the energy distribution in star spectra. Photographic methods. The determination of the intensities of absorption lines in stellar spectra.

"Colorimetry," by K. F. BOTTLINGER (Neubabelsberg): The determination of various color-equivalents. Effective wave-lengths. Color-indices. Color-equivalents. Catalogues. The relations between color-equivalents and other factors.

"Photo-electric Photometry," by H. ROSENBERG (Kiel): Introduction. The construction of photo-electric cells and their properties. Methods for measuring the photo-electric effect. Complete photo-electric installations. Applications of photo-electric photometry in astronomy.

Ibid., Vol. II, Part II: *Foundations of Astrophysics*. Pp. vii+322; Figs. 85. 1931. RM. 49.60; bound, RM. 51.48.

CONTENTS

"Photographic Photometry," by G. EBERHARD (Potsdam): Historical survey. Definitions. The measurement of photographic blackening. The photographic plate and its properties. Photometric methods. The construction of the characteristic curve. Correction for photographic fog. Field corrections. Extinction. The measurement of large intensity intervals. Comparison of photographic catalogues of stellar magnitudes and the international polar sequence. Photographic spectrophotometry.

"Visual Photometry," by W. HASSENSTEIN (Potsdam): The principles of visual photometry developed on the basis of physiological optics. The refractor as a photometric instrument. Methods of measurement. Visual photometers. The estimation of stellar magnitudes.

Ibid., Vol. III, Part I: *Foundations of Astrophysics*. Pp. x+473; Figs. 44. 1930. RM. 66.60; bound, RM. 69.30.

CONTENTS

"Thermal Radiation," by W. WESTPHAL (Berlin): Survey of classical physics. The laws of Stefan-Boltzmann, Wien, and Rayleigh-Jeans. Planck's law. The quantum theory. Measuring instruments and methods. The determination of the radiation constants.

"Thermodynamics of the Stars," by E. A. MILNE (Oxford): Introduction. Survey of the theory of radiative equilibrium. Chromospheric equilibrium. Polytropic gas-spheres. The total energy of a star and the gravitational theory of stellar evolution. The internal equilibrium of a star. The radiative equilibrium of a rotating star.

"The Ionization in the Atmospheres of Stars," by A. PANNEKOEK (Amsterdam): Introduction. The theory of ionization equilibrium. Thermodynamical derivation of the equation of equilibrium. Derivation of the equilibrium equation from statistical mechanics. The mechanism of ionization processes. The structure of stellar atmospheres. Ionization on the sun. Ionization phenomena in star spectra. Spectral classes. The absolute brightnesses of the stars. Emission lines.

"The Principles of the Quantum Theory," by S. ROSSELAND (Oslo): Quantum theory. Wave-mechanics. Pauli's exclusion principle. Fermi-Dirac statistics. The Compton effect. The stellar absorption coefficient.

Ibid., Vol. III, Part II: *Foundations of Astrophysics*. Pp. viii+358; Figs. 131. 1930. RM. 52.92; bound, RM. 55.80.

CONTENTS

"Regularities in Series Spectra," by W. GROTRIAN (Potsdam): The spectra of atoms and of ions possessing a single electron. The spectrum of the hydrogen atom and the spectrum of the ionized helium atom. The spectra of atoms and ions having one valence electron. The spectra of atoms and ions having two valence electrons. The spectra of atoms and ions having three valence electrons. The principal quantum number n. The size and the frequency-difference of the terms.

"The Theory of Multiplet Spectra," by O. LAPORTE (Ann Arbor, Mich.): General discussion of multiplets. Russell-Saunders coupling. Pauli's principle. Quantitative formulae for spectral terms. The Zeeman effect. Intensities and selection rules. Series in complex spectra. Discussion of the various periods and of their spectra.

"Band Spectra," by K. WURM (Potsdam): Band systems and band structure. Electronic terms. Intensities in band spectra. Isotope effects. The spectroscopic determination of the dissociation-energy of molecules.

"Theory of Pulsating Stars," by E. A. MILNE (Oxford): General theory. Pulsation theory. Stability investigations.

Ibid., Vol. IV: *The Solar System*. Pp. viii+501; Figs. 221. 1929. RM. 68.40; bound, RM. 70.92.

CONTENTS

"Radiation and Temperature of the Sun," by W. BERNHEIMER (Vienna): The brightness of the sun. Intensity distribution on the solar disk. Intensity distribution in the solar spectrum. Total radiation of the sun. The solar constant. Variations in the solar constant. The temperature of the sun.

"Solar Physics," by G. ABETTI (Florence): General. Instruments for observing the sun. Visual and photographic observations of the sun's surface and inferences therefrom. Spectroscopic observations of the sun's surface and results derived from them. Theories of the sun.

"Eclipses of the Sun," by S. A. MITCHELL (Charlottesville, Va.): History. The chromosphere. Flash spectrum without a total eclipse. Inter-

pretation of the spectrum of the chromosphere. Photographing the corona. The spectrum of the corona. The light of the corona. Coronal theories. Shadow bands. Einstein's theory.

"The Physical Properties of the Planetary System," by K. GRAFF (Vienna): Survey of the methods of investigation and of the instruments. The inner planets. The outer planets. The moon. The asteroids. The satellites. The zodiacal light.

"Comets and Meteors," by A. KOPFF (Berlin-Dahlem): The orbits of comets and meteors. The physical structure of the comets. Comet tails. Meteors.

Ibid., Vol. V, Part I: *The System of the Stars*. Pp. x+574; Figs. 173. 1932. RM. 96; bound, RM. 99.

CONTENTS

"Classification and Description of Stellar Spectra," by R. H. CURTISS (Ann Arbor, Mich.): The beginning of stellar-spectrum analysis. Early classifications of stellar spectra. Later classifications of stellar spectra. The Harvard classifications. The development of the Draper classification from 1901 to 1924. Presentation of the Draper classification. Comparison of the principal stellar spectral classifications. The Draper classification. Additions and modifications. Additions and redefinitions affecting classes and divisions of the Draper classification. Classifications of stellar spectra not based on estimates of line intensities. Catalogues of stellar spectra. Conclusion.

"Statistical Data on Spectral Types," by F. BECKER (Bonn): Introduction. Statistics on the basis of apparent magnitude and apparent distribution of the stars. Statistics of spectral types with consideration of absolute magnitude and of space-distribution of the stars.

"The Temperatures of the Stars," by A. BRILL (Neubabelsberg): The physical foundations. The temperature problem in astrophysics. Color-temperatures of stars from the shape of the energy-curve. Color-temperatures of stars from the color or from other color-equivalents. The radiation temperature determined from the intensities in certain spectral regions. Summary of observational results. The temperatures of the stars derived from the theory of thermal ionization.

"Luminosities, Colors, Diameters, Densities, Masses of the Stars," by K. LUNDMARK (Lund): Apparent stellar magnitudes. Estimates of magnitudes. Uranometries, visual photometry, photographic photometry,

special problems of stellar photometry. Stellar colors: direct estimates of colors, measured star colors. Absolute magnitudes and some other problems.

Ibid., Vol. V, Part II: *The System of the Stars.* Pp. x+582; Figs. 118; Pls. 2. 1933. RM. 96; bound, RM. 99.

CONTENTS

"Luminosities, Colors, Diameters, Densities, Masses of the Stars [*continued*]," by K. LUNDMARK (Lund): The diameters of the stars. The densities of the stars. The masses of the stars. Appendix I: "Catalogue of Stars Brighter than the Fifth Magnitude." Appendix II: "Catalogue of Stellar Diameters."

"Stellar Clusters," by H. SHAPLEY (Cambridge, Mass.): Introductory survey. Classification, number, and distribution. On the spectral composition of clusters. Variable stars in star clusters. The distribution of stars in globular clusters. The forms of clusters. The transparency of space. The distances and dimensions of clusters. Star clusters in the Magellanic Clouds. Dimensions of the galaxy. Appendix A: "Catalogue of Globular Clusters." Appendix B: "Catalogue of Galactic Clusters."

"The Nebulae," by H. D. CURTIS (Ann Arbor, Mich.): Introduction. The diffuse nebulae. The planetary nebulae. The spirals. Appendices.

"The Milky Way," by B. LINDBLAD (Stockholm): Introduction. The visual appearance of the Milky Way. The photography of the Milky Way. The structure of the Milky Way from visual and photographic observations. The effect of diffuse nebulae upon the appearance of the Milky Way. Astrophysical and statistical results concerning the nature of the Milky Way. The dynamics of the Milky Way.

Ibid., Vol. VI: *The System of the Stars.* Pp. ix+474; Figs. 123. 1928. RM. 59.40; bound, RM. 61.83.

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"The Radial Velocities of the Stars," by K. G. MALMQUIST (Lund): Introduction. Lists and catalogues of radial velocities. The solar motion derived from the radial velocities of the stars. The distribution of stellar velocities as derived from the radial motions. The radial velocities combined with other attributes of the stars.

"Variable Stars," by H. LUDENDORFF (Potsdam): Introduction. Variable stars resembling novae. Variables of the class of R Coronae. Variables of the class of U Geminorum. Mira-type variables. Variables of the

class μ Cephei. Variables of the class RV Tauri. Variables of the class δ Cephei. Variables in star clusters, nebulae, and similar objects. On the relations between different classes of variable stars.

"Novae," by F. J. M. STRATTON (Cambridge): Early history. Distribution of novae. Parallaxes and proper motions of novae. Light-curves and absolute magnitudes. Spectrophotometry and temperature. Spectroscopic history. Nebulosity and novae. Theories.

"Double and Multiple Stars," by F. C. HENROTEAU (Ottawa): Double-star observers and their work. Classification and observation of visual double stars. The orbit of a visual binary star. The visual double stars of known orbits. Some interesting systems. The spectroscopic binaries. The orbit of a spectroscopic binary star. The spectroscopic binaries of known orbits; typical systems. The Algol variables and β Lyrae stars. Stars of the β Canis Majoris type. Statistical and other studies relating to characteristics of double stars. Triple and other multiple systems.

The firm of Julius Springer has just announced that with the appearance of Volume I, their monumental *Handbuch der Astrophysik* is completed, although it is the intention of the publishers and of the editors to prepare a supplementary volume in which recent advances in the various branches of astrophysics may be brought up to date. In spite of its very high price, the *Handbuch* marks an important event in astronomy, because it combines for the first time in one edition a large amount of knowledge accumulated during the last century. The editors are especially to be congratulated upon having brought to a successful conclusion this piece of work, which started in 1928 with the appearance of Volume VI. The total number of pages in all six volumes is 4255 and the price for the bound volumes[1] is 567.75 RM. (approximately $226, at the present rate of exchange). It should be noted that the prices quoted already discount the 10 per cent allowed on all German books published prior to July 1, 1931.

The choice of contributors has, in general, been successful. The work may be regarded as truly international, since not only German astronomers, but also astronomers in England, the United States, Canada, Austria, Holland, Sweden, Norway, and Italy have been invited to cooperate. The languages used are German and English: 2516 pages in the former and 1739 in the latter.

Astronomers will particularly welcome the appearance of the first three volumes, which deal with the foundations of astrophysics. The chapters

[1] As quoted by the *Hirschwaldsche Buchhandlung* for members of the Astronomische Gesellschaft.

on the telescope, on theoretical spectroscopy, on theoretical photometry, on the thermodynamics of the stars, on ionization in stellar atmospheres, on multiplet spectra, and on other subjects are important new monographs on branches of astrophysics which are not readily accessible to the student. There is no doubt that such summaries as those by Milne and by Schoenberg will remain for many years the most complete treatises in their respective fields.

The purely observational volumes are somewhat less complete and are likely to be superseded before long. Even so, they remain valuable summaries of existing knowledge and will often save the investigator the trouble of looking up innumerable individual references.

It is not easy to define what should and what should not be classed under the title of "Astrophysics." Astrophysicists will appreciate the article by Arthur König on the measurement and reduction of direct photographs, although this subject might not necessarily be considered a part of astrophysics. On the other hand, many readers will regret that such important subjects as the determination of stellar parallaxes by the trigonometric method and the determination of proper motions have been excluded because, strictly speaking, they are not branches of astrophysics. No less difficult has been the decision in the field of dynamical astronomy, many branches of which should properly be classified as astrophysics and have, accordingly, found a place in the *Handbuch*.

We regret that there is no chapter on stellar spectroscopy as such. Parts of this subject have been included in Curtiss' account of spectral classification. There are, however, many important data concerning the spectra of the stars which have no direct relation to the practical problem of classifying spectra. Some of these may be found scattered throughout the articles of Curtiss, Lundmark, Pannekoek, and others; but a systematic account would be helpful to the practical spectroscopist. Thus, no detailed account of the observational results of the measurements of contours of absorption lines, of the observational verifications of the ionization theory, of the work of Russell and Adams on stellar spectra, of stellar rotation, or of the Stark effect (although the latter is mentioned in the introductory chapter by Schulz) is contained in the *Handbuch*.

While the *Handbuch* is obviously important to every astronomer, it is clear that such enormous editions cannot appear frequently. Other means must therefore be devised to meet the need of scientific summaries. It is to be hoped that a suitable organization will undertake to supplement the *Handbuch* by publishing from time to time relatively short summarizing articles containing reviews of new developments in various fields.

O. STRUVE

A Textbook of Practical Astronomy. By JASON JOHN NASSAU. New York: McGraw-Hill Book Co., 1932. 8vo. Pp. x+226. Figs. 79. $3.00.

This text will doubtless be welcomed by many who teach astronomy to engineering students. It furnishes a thorough grounding in the fundamentals of practical astronomy without presenting a bewildering variety of methods. Opinions of teachers may differ as to the wisdom of some omissions, but the methods included are those most common in practice and those which illustrate best the fundamentals of the subject.

There are fourteen chapters, of which the first seven furnish the groundwork of general astronomical facts, co-ordinate systems, time and its conversion, the use of the American Ephemeris, corrections to observations, and the adjustment and use of instruments. The remainder of the book is devoted to methods of determining time, latitude, azimuth, and longitude. Chapters on the astronomical transit and the zenith telescope are included.

A critical reader will find in the introductory material an occasional inconsistency, such as that between Figure 1 and the text as to dates of perihelion and aphelion, or a place where accuracy has been sacrificed for brevity, as in the statement concerning the distances and nature of nebulae on page 1.

The chapter on instruments is to be especially commended. All the important adjustments are clearly and concisely explained, and methods of using the instruments are described. For each method of determining latitude, time, or azimuth, a schedule of procedure is given. Illustrative examples are numerous, and forms for the recording of observations and for computation are given at the end of the book. Not much excuse is left for slipshod methods of observing, recording, or computing. The chapters on the astronomical transit and the zenith telescope are not long, but treat their subjects with all necessary thoroughness.

Derivations of formulae are not overemphasized, although none is taken for granted except the three fundamental equations of spherical trigonometry. The figures with which the book is illustrated deserve special mention. The most complex of them are remarkably clear as a result of the simple device of heavy-lining the principal parts so that they stand out from the background of reference circles. They form a welcome contrast to the bewildering mazes of intersecting lines to be found in some earlier texts. The relative position of pole and equator in Figure 22 does some violence to perspective, but the meaning of the figure is clear.

How much importance to attach to local civil time is probably a point on which teachers would disagree widely. Professor Nassau's use of Cleveland Civil Time as an actual clock time will perhaps be objectionable to some. In connection with the determination of latitude by circum-meridian altitudes, the possibility of determining the clock correction is pointed out without sufficient caution as to the inevitable uncertainty of the result. Some hints might also be given concerning choice of method as dictated by the accuracy attainable with the various trigonometric functions, as, for example, a warning against the use of the sine in determining an angle close to 90°.

The points to which exception has been taken are minor ones and should not detract from the genuine merits of the book. It is well written, presents its subject very clearly and concisely, and emphasizes methodical procedure on the part of the student. It would be difficult, indeed, to improve upon the order or method of treatment. The text deserves to be widely adopted by teachers of practical astronomy.

DEAN B. MCLAUGHLIN

Die kosmologischen Probleme der Physik. By ARTHUR HAAS. Leipzig: Akademische Verlagsgesellschaft M.B.H., 1934. Pp. vii+124. RM. 3.80.

This little volume contains a series of lectures presented by its author at the University of Vienna. The first four chapters give the observational foundations of cosmology; there is a brief account of the work of Shapley, Hubble, and others, on the distribution of extra-galactic nebulae, and of Slipher, Hubble, and Humason on the radial velocities of these nebulae. Chapter iii is devoted to estimates of the age of the universe. Chapter iv deals with the radiation in the universe. The fifth chapter is a popular introduction to relativistic cosmology. Chapters vi, vii, and viii deal, respectively, with the work of Einstein and De Sitter, with the universe of Friedman-Lemaître and with the cosmic constants. An Appendix contains the derivation of Friedman's differential equations. The presentation is relatively simple, and the book is an admirable introduction to the study of relativistic cosmology.

O. S.

Hydrodynamique physique, avec applications à la météorologie dynamique. By V. BJERKNES, J. BJERKNES, H. SOLBERG, and T. BERGERON. ("Recueil des conférences—rapports de documentation sur la physique," Vol. 23.) Paris: Les Presses Universitaires de France, 1934. 3 vols. Pp. xvii+864. Fr. 240.

More than forty years ago V. Bjerknes began his work in theoretical hydrodynamics by arranging and expanding the material left incomplete by his father, C. A. Bjerknes. He was thus led immediately to the consideration of the problem of a generalized theory of hydrodynamic fields of force, to which he has since contributed much that is fundamental. In particular, he traced out and made use of two series of analogies between hydrodynamic and electromagnetic fields. In the meantime he had become interested in the application of the theories of fluid motion to meteorology. With the assistance of several of his students, especially J. W. Sandström, he dealt with such problems as the accurate analysis of atmospheric circulation with a view to practical weather forecasting. Working at the University of Leipzig from 1913 until 1917, and since then at Bergen and Oslo in Norway, he has trained an active group of research men, including his collaborators in the present volume. Among their numerous contributions should be mentioned particularly Bjerknes's well-known polar-front theory of the formation of cyclones.

Bjerknes and his school have thus built up an extensive system of physical hydrodynamics in which the classical theories, which were developed for the most part by considering ideally homogeneous and incompressible fluids, are modified to permit of the treatment of actual geophysical conditions, where temperature, pressure, and density are variables.

The present treatise sets forth this system in detail. After an introductory outline of the vector calculus, used consistently throughout the book, the authors set up the fundamental equations of motion and of equilibrium, placing emphasis upon the differences in the physical interpretations of the Lagrangian and Eulerian approaches. This care in making physical explanations an integral part of the extensive mathematical developments is characteristic of the entire work and forms one of its most useful features.

The fifth and sixth chapters are devoted to comparisons of the forces in hydrodynamic fields with those of electric and magnetic fields. With the basic equations established, the following eight chapters deal with the method of perturbations developed by Bjerknes for handling cases of more

complicated motion. Thus the development of cyclones is worked out as a perturbation of rectilinear currents upon the rotating earth.

The last part of the work comprises a distinct section of some one hundred and fifty pages, written chiefly by J. Bjerknes and Bergeron, on applications of the general theories to practical meteorology.

Taken as a whole, the treatise represents an individuality of viewpoint which results in a well-unified presentation. However, this same characteristic involves a tendency to neglect the contributions of other men, and to cover only cursorily those branches of the subject with which the authors had not been directly concerned in their own research. For example, as H. Ertel[1] has pointed out in his review of the German edition,[2] the subject of turbulence is dismissed in a page or so.

The astrophysicist will regret that solar hydrodynamics, on which V. Bjerknes contributed a preliminary paper in 1926, has not been developed further, but in a paragraph of the historical outline at the end of the work some suggestions are made which may indicate a line of attack to specialists wishing to go into the subject.

P. C. KEENAN

Resonance Radiation and Excited Atoms. By ALLAN C. G. MITCHELL and MARK W. ZEMANSKY. New York: Macmillan Co.; Cambridge: University Press, 1934. Pp. xvi+338. $5.00.

The need for such a book as this has become acute in the last few years. Workers in the many branches of chemistry, physics, and astrophysics in which the spectra of gases play an important part must continually ask how radiation transfers part of its energy to particles encountered in its path, and how the light which is then emitted by these excited atoms is modified by collisions and by absorptions on the part of neighboring atoms. Although the theory and observations bearing on these questions had their start near the beginning of the century in the investigations by Voigt and by Wood, the rapid development of the subject has taken place only within the last decade, following the establishment of the quantum theory. The mass of published material expressing this progress has required unification by a discussion embracing the whole field. Fortunately,

[1] *Naturwissenschaften*, **22**, 175, 1934.

[2] *Physikalische Hydrodynamik mit Anwendung auf die dynamische Meteorologie.* Berlin: Julius Springer, 1933. RM. 66–; geb. RM. 69–.

the task has been undertaken by two authors who have contributed broadly to the subject by their own research.

Mitchell and Zemansky give first a general description of the physical and chemical effects connected with radiation. They then take up the details under three divisions: absorption lines and measurements of lifetimes of the resonance states, collision processes involving excited atoms, and the polarization of resonance radiation.

In the more technical sections the authors summarize the experimental procedure and related theory, but they are concerned with the results rather than with detailed derivations. The work is therefore to be regarded not as a complete treatise but as an outline to be used in conjunction with the original papers. This is a disadvantage to students in the case of such topics as the theory of the absorption coefficient and Milne's treatment of the diffusion of radiation, where the sources are journals not always readily accessible. However, the authors evidently considered it desirable to make the book fairly compact, while supplying abundant references (some three hundred and forty technical papers are cited). The data treated are not merely thrown together into an impressive pile, but are analyzed with discrimination, the equations being transformed into one system of notation and the essential conclusions made to stand out in clear relief.

To research workers the comprehensive tables of lifetimes of excited states, effective cross-sections for various collisional processes, and values of integrals involved in formulae for total absorptions will be especially useful.

No mention is made of astronomical instances of resonance phenomena. The possibility that resonance may play a significant part in fixing the intensities of emission lines in the spectra of the chromosphere, planetary nebulae, and interstellar gases is of great theoretical interest, and it might seem at first sight that some account of these points should be given in any general work on the subject. However, it must be admitted that the evidence available now is vague; in no case has it been possible to assign to resonance its precise importance as a factor in excitation. For this reason it may be just as well that a critical discussion of the matter be deferred until later.

P. C. KEENAN

Handbuch der Spektroskopie, Vol. 8, Part I. By H. KAYSER and H. KONEN. Leipzig: S. Hirzel, 1932. Pp. iv+654. Unbound, RM. 67.50.

This work is of the first importance to the practical astrophysicist. Revised data are collected and edited for nineteen elements: *Ag*, *Al*, *A*, *As*, *Au*, *B*, *Ba*, *Be*, *Bi*, *Br*, *C*, *Ca*, *Cd*, *Ce*, *Cl*, *Co*, *Cr*, *Cs*, and *Cu*. The discussion is based on material published in the period 1922–1932, inclusive, and includes a section on the simpler carbon molecules, in addition to the atomic spectra.

W. W. MORGAN

NOTICE TO CONTRIBUTORS

At the request of several contributors, the *Astrophysical Journal* has adopted the following list of abbreviations for use in footnotes. Contributors who prefer to use complete titles may continue to do so in the future.

A.J.	Astronomical Journal
Ap. J.	Astrophysical Journal
A.N.	Astronomische Nachrichten
Ann. d. Phys.	Annalen der Physik
B.A.	Bulletin Astronomique
B.A.A.	British Astronomical Association
B.A.N.	Bulletin of the Astronomical Institutes of the Netherlands
Beob. Zirk.	Beobachtungs-Zirkular der Astronomischen Nachrichten
C.R.	Comptes Rendus
Dom. Ap. Obs.	Dominion Astrophysical Observatory
Harvard Ann.	Annals of the Harvard College Observatory
Harvard Bull.	Bulletins of the Harvard College Observatory
Harvard Circ.	Circulars of the Harvard College Observatory
Lick Obs. Bull.	Lick Observatory Bulletin
Lund. Medd.	Meddelanden från Lunds Astronomiska Observatorium
Mem. R A.S.	Memoirs of the Royal Astronomical Society
M.N.	Monthly Notices of the Royal Astronomical Society
Mt. W. Comm.	Communications to the National Academy of Science from the Mount Wilson Observatory
Mt. W. Contr.	Contributions from the Mount Wilson Observatory
Nature	Nature
Observatory	Observatory [the periodical]
Phil. Mag.	Philosophical Magazine
Phys. Rev.	Physical Review
Pop. Astr.	Popular Astronomy
Proc. Nat. Acad.	Proceedings of the National Academy of Science
Proc. R. Soc.	Proceedings of the Royal Society of London
Pub. A.A.S.	Publications of the American Astronomical Society

Pub. A.S.P.	Publications of the Astronomical Society of the Pacific
Science	Science [the periodical]
Trans. I.A.U.	Transactions of the International Astronomical Union
V.J.S.	Vierteljahrsschrift der Astronomischen Gesellschaft
Zs. f. Ap.	Zeitschrift für Astrophysik

Acad.	—	Academy	Mt. W.	—	Mount Wilson
Amer.	—	American	Obs.	—	Observatory
Ann.	—	Annals	Opt.	—	Optical
Bull.	—	Bulletin	Phil.	—	Philosophical
Bur.	—	Bureau	Phys.	—	Physical
Chem.	—	Chemical	Proc.	—	Proceedings
Circ.	—	Circular	Pub.	—	Publications
Contr.	—	Contributions	R.	—	Royal
Dom.	—	Dominion	Sci.	—	Science
f.	—	für, for	Sitz.	—	Sitzungsberichte
Instr.	—	Instruments	Stand.	—	Standards
J.	—	Journal	Trans.	—	Transactions
Lab.	—	Laboratory	U.	—	University
Mag.	—	Magazine	Verh.	—	Verhandlungen
Medd.	—	Meddelanden	Veröff.	—	Veröffentlichungen
Mem.	—	Memoirs	Zs.	—	Zeitschrift
Mitt.	—	Mitteilungen			

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